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Discrete Optimization

The multi-compartment vehicle routing problem with flexible compartment sizes

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ABSTRACT

In this paper, a capacitated vehicle routing problem is discussed which occurs in the context of glass waste collection. Supplies of several different product types (glass of different colors) are available at customer locations. The supplies have to be picked up at their locations and moved to a central depot at minimum cost. Different product types may be transported on the same vehicle, however, while being transported they must not be mixed. Technically this is enabled by a specific device, which allows for separating the capacity of each vehicle individually into a limited number of compartments where each compartment can accommodate one or several supplies of the same product type. For this problem, a model formulation and a variable neighborhood search algorithm for its solution are presented. The performance of the proposed heuristic is evaluated by means of extensive numerical experiments. Furthermore, the economic benefits of introducing compartments on the vehicles are investigated.

1. Introduction

The vehicle routing problem, which will be discussed in this paper, is a variant of the classic capacitated vehicle routing problem (CVRP; for surveys see Golden, Raghavan, & Wasil, 2008; Laporte, 2009; or Toth & Vigo, 2014) and occurs in the context of glass waste collection in Germany. Glass waste has to be recycled by law and is used as a raw material for the production of new glass products. It has to be taken to recycling stations by the consumers where it is disposed into different containers according to the color of the waste (usually colorless, green and brown glass). Colors are kept separated because the production of new glass products is less cost-intensive if the glass waste is not too inhomogeneous with respect to its color. Trucks, which are located at a depot of a recycling company, pick up the glass waste from the recycling stations. Since they possess a relatively large loading capacity, they can call at several recycling stations before they have to return to the depot. Recent truck models are equipped with a special device which allows for introducing bulkheads in predefined positions of the loading space such that it can be split into different compartments and, thus, enabling transportation of glass waste with different colors on the same truck without mixing the colors on the tour. This gives rise to the question how the tours of the trucks should be designed given the availability of a device of this kind.

The problem under discussion can be classified as a multi-compartment vehicle routing problem (MCVRP). However, it is different from the ones previously discussed in the literature with respect to the following properties:

- The size of each compartment is not fixed in advance but can be determined individually for each vehicle/each tour.
- The size of the compartments can only be varied discretely, i.e. the walls separating the compartments from each other can only be introduced in specific, predefined positions.
- The number of compartments, into which the capacity of a vehicle is divided, can be identical to the number of product types (glass waste types) but can also be smaller.

Consequently, not only the vehicle tours have to be determined, but it has also to be decided for each vehicle/tour (i) into how many compartments the vehicle capacity should be divided, (ii) what the size of each compartment should be, and (iii) which product type should be assigned to each compartment.

The problem is NP-hard, since it is a generalization of the CVRP (see, for example, Toth & Vigo, 2014). By application of a mathematical model-based exact solution approach, we were only able to solve problem instances with a limited size to optimality. Therefore, a heuristic, namely a variable neighborhood search (VNS), has been developed and will be presented. According to the best of our

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knowledge, this is the first method which has been proposed for this problem so far. We will further analyze what the economic benefits are which stem from the introduction of flexibly sizable compartments.

The remainder of this paper is organized as follows. Section 2 presents a formal definition and a mathematical formulation of the problem. The relevant literature related to the MCVRP is discussed in Section 3. In Section 4, the proposed variable neighborhood search algorithm is introduced. Extensive numerical experiments have been performed in order to evaluate the mathematical model and the VNS. The design of these experiments and the corresponding results are presented in Section 5. Finally, the main findings are summarized and an outlook on future research is given in Section 6.

2. Problem description and formulation

The multi-compartment vehicle routing problem with flexible compartment sizes (MCVRP-FCS) can be formulated as follows: Let an undirected, weighted graph $G = (V, E)$ be given which consists of a vertex set $V = \{0, 1, \dots, n\}$, representing the location of the depot ($\{0\}$) and the locations of n customers ($\{1, \dots, n\}$), and an edge set $E = \{(i, j): i, j \in V, i < j\}$, representing the edges which can be traveled between the different locations. To each of these edges, a non-negative cost c_{ij}, $(i, j) \in E$, is assigned. It is assumed that all of these costs satisfy the triangle inequality.

Further, let a set P of product types be given. At each vertex (except for the depot) exists a non-negative supply $s_{ip} (i \in V\backslash\{0\}, p \in P)$ of each of the product types. The supplies have to be collected at their locations and transported to the depot without the product types being mixed. A location may be visited several times in order to pick up different product types. However, if being picked up, each supply has to be loaded in total. In other words, a split collection of a single supply is not permitted.

For the purpose of transportation, a set K of homogeneous vehicles is available, each equipped with a total capacity Q. Individually for each vehicle $k \in K$, the total capacity Q can be divided into a limited number $\hat{m}$ of compartments, $\hat{m} \le |P|$, which allows for loading products of different types on a single vehicle while keeping them separated during transportation. The size of the compartments can be varied discretely in equal step sizes, i.e. each compartment size, but also the total vehicle capacity Q, is an integer multiple of a basic compartment unit size q^{unit}. Let the set of these multiples be denoted by $M = \{0, 1, 2, \dots, m^{\max}\}$ where $m^{\max} = Q/q^{\text{unit}}$. Then $q_m = \frac{1}{m^{\max}} \cdot m$, $m \in M$, denotes a compartment size relative to the total capacity Q which consists of m ($m \in M$) multiples of the basic compartment unit size q^{unit}. To illustrate these aspects, we introduce a small example in which the vehicle capacity Q amounts to 200 units and the basic compartment unit size q^{unit} to 10 units. Hence, only compartment sizes of 10, 20, 30,..., 200 units or 5 percent, 10 percent, 15 percent,..., 100 percent of the vehicle capacity can be selected. Accordingly, $m^{\max}$ is equal to 20 and a compartment with $m = 7$ corresponds to a relative compartment size of $q_m = \frac{1}{m^{\max}} \cdot m = \frac{1}{20} \cdot 7 = 0.35$, i.e. 35 percent of the vehicle capacity. It is important to note that the set of potential compartment configurations is identical for all vehicles. However, the actual configuration in a particular solution might be different for each vehicle.

What has to be determined is a set of vehicle tours, an assignment of product types to the vehicles and the sizes of the corresponding compartments such that all supplies are collected, that the capacity of none of the used vehicles is exceeded, and that the total cost of all edges to be traveled is minimized.

This problem involves the following partial decisions to be made simultaneously:

- assignment of product types to each of the vehicles
  (this decision determines which product types can be collected by each vehicle);
- determination of the size of each compartment
  (this decision fixes for each vehicle how its total capacity is split into compartments);
- assignment of supplies to each of the vehicles
  (this decision implicitly includes an assignment of locations to vehicles);
- sequencing of the locations for each of the vehicles
  (this decision determines for each vehicle in which sequence the assigned locations are to be visited).

We note that every vehicle routing problem involves decisions of the last two types, while the first and the second one define the uniqueness of the MCVRP-FCS.

In order to formulate a mathematical model for the MCVRP-FCS, we introduce the following four types of variables:

$$u_{ipk} = \begin{cases} 1, & \text{if supply of product type } p \text{ at location } i \text{ is collected by vehicle } k, \\ 0, & \text{otherwise,} \end{cases} \quad i \in V\backslash\{0\}, p \in P, k \in K;$$

$$x_{ijk} = \begin{cases} 2, & \text{if } i = 0 \text{ and edge } (i, j) \text{ is used twice by vehicle } k, \\ 1, & \text{if edge } (i, j) \text{ is used once by vehicle } k, \\ 0, & \text{otherwise,} \end{cases} \quad i, j \in V : i < j, k \in K;$$

$$y_{pkm} = \begin{cases} 1, & \text{if size } q_m \text{ is selected for product type } p \text{ in vehicle } k, \\ 0, & \text{otherwise,} \end{cases} \quad p \in P, k \in K, m \in M;$$

$$z_{ik} = \begin{cases} 1, & \text{if location } i \text{ is visited by vehicle } k, \\ 0, & \text{otherwise,} \end{cases} \quad i \in V, k \in K.$$

The objective function and the constraints of the model can then be formulated as follows:

$$\min \sum_{(i,j) \in A} \sum_{k \in K} c_{ij} x_{ijk} \tag{1}$$

$$\sum_{k \in K} u_{ipk} = 1 \quad \forall i \in V\backslash\{0\}, p \in P : \; s_{ip} > 0 \tag{2}$$

$$u_{ipk} \le z_{ik} \quad \forall i \in V\backslash\{0\}, p \in P, k \in K \tag{3}$$

$$z_{ik} \le z_{0k} \quad \forall i \in V\backslash\{0\}, k \in K \tag{4}$$

$$\sum_{\substack{j \in V: \\ j > 0}} \sum_{k \in K} x_{0jk} \le 2|K| \tag{5}$$

$$\sum_{\substack{j \in V: \\ i < j}} x_{ijk} + \sum_{\substack{j \in V: \\ j < i}} x_{jik} = 2 z_{ik} \quad \forall i \in V, k \in K \tag{6}$$

$$\sum_{p \in P} \sum_{m \in M} y_{pkm} \le \hat{m} \quad \forall k \in K \tag{7}$$

$$\sum_{p \in P} \sum_{m \in M} q_m y_{pkm} \le 1 \quad \forall k \in K \tag{8}$$

$$\sum_{i \in V\backslash\{0\}} s_{ip} u_{ipk} \le Q \sum_{m \in M} q_m y_{pkm} \quad \forall p \in P, k \in K \tag{9}$$

$$\sum_{i \in S} \sum_{\substack{j \in S: \\ i < j}} x_{ijk} \le |S| - 1 \quad \forall k \in K, S \subseteq V\backslash\{0\} : |S| > 2 \tag{10}$$