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Interfaces with Other Disciplines

Identification of the anchor points in FDH models

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ABSTRACT

This paper investigates the anchor points in nonconvex Data Envelopment Analysis (DEA), called Free Disposal Hull (FDH), technologies. We develop the concept of anchor points under various returns to scale assumptions in FDH models. A necessary and sufficient condition for characterizing the anchor points is provided. Since the set of anchor points is a subset of the set of extreme units, a definition of extreme units in non-convex technologies as well as a new method for obtaining these units are given. Finally, a polynomial-time algorithm for identification of the anchor points in FDH models is provided. Obtaining both extreme units and anchor points is done via calculating only some ratios, without solving any mathematical programming problem.

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1. Introduction

The boundary of the Production Possibility Set (PPS) (production technology) plays a crucial role in performance evaluation and productivity analysis. There are several tools in the literature to estimate the PPS and its boundary. One of the most important and widely-used approaches is Data Envelopment Analysis (DEA), which is a non-parametric Linear Programming-based technique. DEA was first introduced by Charnes, Cooper, and Rhodes (1978) and was developed by many scholars; see e.g. Cook and Seiford (2009), Cooper, Seiford, and Tone (2007), Emrouznejad, Parker, and Tavares (2008), and Hatami-Marbini, Emrouznejad, and Tavana (2011) for some reviews.

An important class of DEA technologies is that of FDH models. These models, which have been first presented by Deprins, Simar, and Tulkens (1984), evaluate the Decision Making Units (DMUs) considering the closest inner approximation of the true strongly disposable (but possibly non-convex) technology. FDH models have been studied by many scholars, including Tulkens (1993), Kerstens and Vanden Eeckaut (1999), Cherchye, Kuosmanen, and Post (2000, 2001), Podinovski (2004), Leleu (2006), Briec, Kerstens, and Vanden Eeckaut (2004), Briec and Kerstens (2006), Soleimani-damaneh, Jahanshahloo, and Reshadi (2006), Soleimani-damaneh and Reshadi (2007), and Soleimani-damaneh and Mostafaee (2009). See also, Kerstens and Woestyne (2014) for a recent review of the solution methods in FDH models.

Studying the boundary points of DEA technologies is a very important issue in performance analysis and this leads to theoretical and practical observations which are useful in studying the structure of efficiency and inefficiency. Charnes, Cooper, and Thrall (1991) classified Decision Making Units (DMUs) to six classes based upon their efficiency situation, the dimension of dual solution set, and the positivity of the optimal multipliers. In another work in the economics literature, Färe, Grosskopf, and Lovell (1983) classified the boundary points of production technologies to three classes, isoquant, weak efficient, and efficient. They utilized this classification to address the question, what is technical inefficiency and where does it come from?

An important set of boundary points which are defined according to the positivity of optimal multipliers is that of anchor points.Thanassoulis and Allen (1998) used the concept of these points, at first, for the generation of unobserved DMUs and extending the DEA frontier. Bougnol and Dulá (2009) defined these points formally as production possibilities which give the transition from the Pareto-efficient frontier to the free-disposability portion of the boundary of the PPS. Rouse (2004) utilized this notion for identifying prices for health care services. Bougnol and Dulá (2009) used the geometrical properties of the anchor points to design and test an algorithm for their identification. Thanassoulis, Kortelainen, and Allen (2012) provided another method for identifying the anchor points based upon the radial efficiency scores and slack variables at the optimal solution of envelopment models. They have used this concept for improving envelopment under multiple inputs and outputs in a VRS technology. In a recently published paper, Mostafaee and Soleimani-damaneh (2014) presented an algorithm for identification

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of the anchor points utilizing sensitivity analysis techniques. See also Bougnol (2001) and Allen and Thanassoulis (2004) for more details about the notion and applications of the anchor points.

All of the above-mentioned works have been done on anchor points under convex technologies. In this paper, we study the anchor points in (nonconvex) FDH models. To the best of our knowledge, it is the first work on anchor points in nonconvex technologies. We define the extreme unit and anchor point notions in nonconvex technologies. Since the first step for obtaining the anchor points is obtaining the extreme units, a ratio-based technique for determining the extreme units in FDH models is presented. Necessary and sufficient conditions for characterizing the anchor points are established, and utilizing them, a ratio-based technique is given for determining the anchor points. Both given ratio-based techniques are polynomial-time and they work without solving any mathematical programming problem.

The rest of the paper unfolds as follows: In Section 2, some preliminaries are provided. Section 3 is devoted to defining and identifying the extreme units in FDH technologies. In Section 4, the anchor point notion in FDH technologies is defined and after presenting a characterization, a polynomial-time algorithm for identification of these points is presented. In addition to the theoretical results, some numerical examples are given. Eventually, Section 5 contains a short conclusion.

2. Preliminaries

Suppose that we have a set of n peer DMUs, $\{\text{DMU}_j,\ j\in J=\{1,2,\ldots,n\}\}$, such that each DMU_j produces multiple outputs $y_{rj}>0$ $(r=1,\ldots,s)$ by utilizing multiple inputs $x_{ij}>0$ $(i=1,\ldots,m)$. We assume that there is not any duplicated DMU. Furthermore, let $x_j=(x_{1j},\ldots,x_{mj})^T$ and $y_j=(y_{1j},\ldots,y_{sj})^T$. Superscript "$T$" stands for transpose.

A unified algebraic representation of FDH technologies under different Returns to Scale (RTS) assumptions can be expressed as follows:

$$P^{\Gamma} = \left\{(x,y): \sum_{j\in J}\lambda_j x_j \le x, \sum_{j\in J}\lambda_j y_j \ge y \ge 0, \lambda_j=\delta\omega_j;\ j\in J, \sum_{j\in J}\omega_j=1, \omega\in(\{0,1\})^n,\ \delta\in\Gamma\right\},$$

where Γ, depending on the RTS assumption of the reference technology, is

$$\Gamma^{\text{VRS}}=\{\delta\mid\delta=1\}, \tag{1}$$
$$\Gamma^{\text{CRS}}=\{\delta\mid\delta\ge 0\}, \tag{2}$$
$$\Gamma^{\text{NIRS}}=\{\delta\mid 0\le\delta\le 1\}, \tag{3}$$
$$\Gamma^{\text{NDRS}}=\{\delta\mid\delta\ge 1\}. \tag{4}$$

Here, VRS, CRS, NIRS, and NDRS stand for Variable, Constant, Nonincreasing, and Nondecreasing RTS, respectively.

Considering DMU_o $(o\in J)$ as the unit under assessment, the input-oriented and output-oriented FDH radial efficiency measures of $\text{DMU}_o=(x_o,y_o)$ are obtained by solving the following mixed-integer nonlinear programming problems, respectively:

$$\begin{aligned}\theta_o^{\Gamma} = \ & \min\ \theta \\ & s.t. \sum_{j\in J}\lambda_j x_j \le \theta x_o, \\ & \quad \sum_{j\in J}\lambda_j y_j \ge y_o, \\ & \quad \lambda_j=\delta\omega_j,\ \omega_j\in\{0,1\};\ j\in J, \\ & \quad \delta\in\Gamma,\ \sum_{j\in J}\omega_j=1,\end{aligned} \tag{5}$$

$$\begin{aligned}\varphi_o^{\Gamma} = \ & \max\ \varphi \\ & \text{s.t.} \sum_{j\in J}\lambda_j x_j \le x_o, \\ & \quad \sum_{j\in J}\lambda_j y_j \ge \varphi y_o, \\ & \quad \lambda_j=\delta\omega_j,\ \omega_j\in\{0,1\};\ j\in J, \\ & \quad \delta\in\Gamma,\ \sum_{j\in J}\omega_j=1,\end{aligned} \tag{6}$$

where $\Gamma\in\{\text{CRS},\text{VRS},\text{NIRS},\text{NDRS}\}$.

The DMU_o is called input-oriented FDH-efficient (corresponding to the Γ set used) if $\theta_o^{\Gamma}=1$. Also, DMU_o is called output-oriented FDH-efficient (corresponding to the Γ set used) if $\varphi_o^{\Gamma}=1$.

The above models and Γ-technologies have been proposed by Kerstens and Vanden Eeckaut (1999). A linear version of these models has been addressed by Podinovski (2004). These models and technologies have been utilized by Soleimani-damaneh et al. (2006), Soleimani-damaneh and Reshadi (2007), and Soleimani-damaneh and Mostafaee (2009) for determining RTS under nonconvex production technologies.

Obtaining FDH efficiency scores using Models (5) and (6) requires solving linear/nonlinear mixed-integer programming problems. The following proposition shows that these models can be solved by calculating only some simple ratios. The proof of this proposition comes from the discussions provided by Soleimani-damaneh et al. (2006), Soleimani-damaneh and Reshadi (2007), and Kerstens and Woestyne (2014), and is hence omitted.

Proposition 2.1. *For $o, j\in J$, define*

$$\lambda_{jo}=\max_r\left\{\frac{y_{ro}}{y_{rj}}\right\}, \qquad \lambda^{jo}=\min_i\left\{\frac{x_{io}}{x_{ij}}\right\}.$$

We have

$$\theta_o^{\Gamma}=\begin{cases}\min\limits_{j\in J:y_j\ge y_o}\left\{\max\limits_i\left\{\dfrac{x_{ij}}{x_{io}}\right\}\right\}, & \text{for } \Gamma=\text{VRS}\\ \min\limits_{j\in J}\left\{\max\limits_i\left\{\dfrac{x_{ij}\lambda_{jo}}{x_{io}}\right\}\right\}, & \text{for } \Gamma=\text{CRS}\\ \min\limits_{j\in J:\lambda_{jo}\le 1}\left\{\max\limits_i\left\{\dfrac{x_{ij}\lambda_{jo}}{x_{io}}\right\}\right\}, & \text{for } \Gamma=\text{NIRS}\\ \min\limits_{j\in J:\lambda_{jo}\ge 1}\left\{\max\limits_i\left\{\dfrac{x_{ij}\lambda_{jo}}{x_{io}}\right\}\right\}, & \text{for } \Gamma=\text{NDRS}\end{cases}$$

$$\varphi_o^{\Gamma}=\begin{cases}\max\limits_{j\in J:x_j\le x_o}\left\{\min\limits_r\left\{\dfrac{y_{rj}}{y_{ro}}\right\}\right\}, & \text{for } \Gamma=\text{VRS}\\ \max\limits_{j\in J}\left\{\min\limits_r\left\{\dfrac{y_{rj}\lambda^{jo}}{y_{ro}}\right\}\right\}, & \text{for } \Gamma=\text{CRS}\\ \max\limits_{j\in J:\lambda^{jo}\le 1}\left\{\min\limits_r\left\{\dfrac{y_{rj}\lambda^{jo}}{y_{ro}}\right\}\right\}, & \text{for } \Gamma=\text{NIRS}\\ \max\limits_{j\in J:\lambda^{jo}\ge 1}\left\{\min\limits_r\left\{\dfrac{y_{rj}\lambda^{jo}}{y_{ro}}\right\}\right\}, & \text{for } \Gamma=\text{NDRS}\end{cases}$$

From Proposition 2.1, it can be seen that the FDH efficiency scores can be obtained by calculating only some ratios. Calculating these ratios is polynomial-time computationally; see Theorem 3 in Soleimani-damaneh and Reshadi (2007).

3. Extreme units in FDH technologies

Since the set of anchor points is a subset of the set of extreme units, studying (identifying) the extreme units plays a crucial role in

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