European Journal of Operational Research 245 (2015) 392–401

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

A matheuristic for the Team Orienteering Arc Routing Problem

Claudia Archetti [a,*], Ángel Corberán [b], Isaac Plana [c], José Maria Sanchis [d], M. Grazia Speranza [a]

[a] Dipartimento di Economia e Management, Università di Brescia, Italy
[b] Departamento de Estadística e Investigación Operativa, Universidad de Valencia, Spain
[c] Departamento de Matemáticas para la Economía y la Empresa, Universidad de Valencia, Spain
[d] Departamento de Matemática Aplicada, Universidad Politécnica de Valencia, Spain

ARTICLE INFO

Article history:
Received 5 March 2014
Accepted 12 March 2015
Available online 16 March 2015

Keywords:
Team Orienteering Problem
Arc routing problem
Routing problems with profits
Matheuristic

ABSTRACT

In the Team Orienteering Arc Routing Problem (TOARP) the potential customers are located on the arcs of a directed graph and are to be chosen on the basis of an associated profit. A limited fleet of vehicles is available to serve the chosen customers. Each vehicle has to satisfy a maximum route duration constraint. The goal is to maximize the profit of the served customers. We propose a matheuristic for the TOARP and test it on a set of benchmark instances for which the optimal solution or an upper bound is known. The matheuristic finds the optimal solutions on all, except one, instances of one of the four classes of tested instances (with up to 27 vertices and 296 arcs). The average error on all instances for which the optimal solution is available is 0.67 percent.

1. Introduction

In arc routing problems customers are located on arcs. The basic problems of this class are the Chinese Postman Problem (CPP), where all edges or arcs have to be traversed, and the Rural Postman Problem (RPP), where only some edges or arcs are required to be traversed, and routes of minimum cost have to be identified. For a formal definition of the CPP and RPP the reader is referred to the book edited by Dror (2000). There are several applications of arc routing problems where the edges or arcs to be traversed are not given and have instead to be selected on the basis of a profit. In fact, for any arc routing problem with given customers to traverse a version where customers have to be chosen is likely to have interesting applications. Typical applications of arc routing problems include road maintenance, garbage collection, and mail delivery. In all these cases, if it is not possible to traverse all customers in a day because vehicles, people or time are not sufficient, one has to choose the most valuable customers to serve.

In the routing literature where customers are located on vertices, the problems where the customers have to be selected on the basis of their profit are called *routing problems with profits* (see Archetti, Speranza, & Vigo, 2014; Feillet, Dejax, & Gendreau, 2005a; Vansteenwegen, Souffriau, & Van Oudheusden, 2011). The arc routing counterpart of this class is called *arc routing problems with profits*. A recent survey on arc routing problems with profits can be found in Archetti and Speranza (2014a). We refer the reader to this survey for the description of the problems that have been studied till now in the literature. It is important to mention that, as pointed out in Archetti and Speranza (2014a), the number of papers dealing with arc routing problems with profits is still very limited, especially if compared with the node routing counterpart. More specifically, the contributions related to the study of problems dealing with more than one vehicle are very few. In fact, the only problems that have been studied (apart from the TOARP that is studied in this paper) are the Profitable Arc Tour Problem (PATP) and the Undirected Capacitated Arc Routing Problem with Profits (UCARPP). The PATP was introduced in Feillet, Dejax, and Gendreau (2005b) and is the problem of finding a set of routes maximizing the difference between the total collected profit and the travel cost such that the travel time of each route does not exceed a given threshold. No limit on the number of routes is given. In Feillet et al. (2005b) an exact algorithm is proposed based on the branch-and-price scheme while in Euchi and Chabchoub (2011) different heuristic algorithms are developed. The UCARPP was introduced in Archetti, Feillet, Hertz, and Speranza (2010) and is the problem of finding the set of routes that maximizes the total collected profit in such a way that each route satisfies capacity and maximum duration constraints. In Archetti et al. (2010) an exact algorithm and different heuristics are proposed. More recently, the same problem was addressed in Zachariadis and Kiranoudis (2011) where a new and effective heuristic algorithm is presented.

A well studied problem in the class of routing problems with profits is the Team Orienteering Problem (TOP), where a fleet of

* Corresponding author. Tel.: +39 0302988587; fax: +39 0302400925.
E-mail addresses: claudia.archetti@unibs.it (C. Archetti), angel.corberan@uv.es (Á. Corberán), isaac.plana@uv.es (I. Plana), jmsanchis@mat.upv.es (J.M. Sanchis), grazia.speranza@unibs.it (M.G. Speranza).

http://dx.doi.org/10.1016/j.ejor.2015.03.022
0377-2217/

uncapacitated vehicles is available, with a time duration constraint on each route, and the problem is to select a set of customers to maximize the total profit of the customers served. In this paper we study the arc routing counterpart of the TOP, the Team Orienteering Arc Routing Problem (TOARP), that was introduced in Archetti, Corberán, Plana, Sanchis, and Speranza (2014). In the TOARP, customers with an associated profit are located on arcs and a fleet of vehicles with a time duration constraint on each route is given. The problem consists in choosing the customers and in designing the routes in such a way that the collected profit is maximized. An interesting application of the TOARP is in truck-load transportation, where customers place orders consisting of requests of transportation services from an origin to a destination. Each transportation service requires a full truck going from the corresponding origin location to the corresponding destination. These services can be represented as arcs of a graph that have to be traversed in order to satisfy the corresponding customer requests. Some of the customers are to be served whereas others may be postponed or not served at all. For example, the service of the least profitable customers may be outsourced. In Archetti et al. (2014) an extended polyhedral study for the TOARP is presented. The proposed branch-and-cut algorithm solves instances with up to 100 vertices, 800 arcs, and 4 vehicles to optimality and makes use of the solutions provided by the heuristic described in this paper.

We address the heuristic solution of the TOARP by means of an algorithm that combines a tabu search, to escape from local optima, a diversification phase and the optimal solution of integer linear programming (ILP) models to intensify the search in some areas of the solution space. The combination of a heuristic or metaheuristic scheme with mixed integer linear programming (MILP) models has been recently explored by several authors. A survey is presented by Ball (2011) for combinatorial optimization problems in general while, more recently, in Archetti and Speranza (2014a) a survey focused on routing problems can be found. These heuristics are named in different ways. In the survey (Ball, 2011) they are simply called *heuristics based on mathematical programming*, in other cases (see Montoya-Torres, Juan, Huatuco, Faulin, & Rodriguez-Verjan, 2012) they go under the generic name of *hybrid heuristics*. The name *matheuristic* was created ad hoc for this class of heuristics (see Maniezzo, Stützle, & Voß, 2010) and this is the name we will use in this paper. The main contribution of this paper is the design of a matheuristic for the TOARP where we exploit the benefit of using ILP models in the searching phase. We developed a solution algorithm which combines heuristic operators with the exact solution of different ILP models. These models can be easily implemented and are powerful tools to intensify the search around a promising solution, while typically standard heuristic operators are not as effective or have to be adapted to the problem structure, thus leading to very specialized solution methods. We performed tests on a large set of benchmark and randomly generated instances. The matheuristic finds the optimal solution on 78 percent of the instances for which the optimal solution is known and provides an average error with respect to the optimal solution of 0.67 percent.

The paper is organized as follows. In Section 2 we introduce the TOARP, whereas in Section 3 we describe the general scheme of the matheuristic and its components. The computational results are presented and discussed in Section 4.

2. The Team Orienteering Arc Routing Problem

The TOARP is defined on a directed graph. In general, the graph is not complete. A numerical value representing the traversal cost or travel time is associated with each arc. Only some of the arcs represent customers. Some customers have to be served, and are called *required*, whereas some others may be served if beneficial. A profit is associated with each customer of the latter set. We call these customers *profitable*. A limited fleet of vehicles is available. Each vehicle starts its route at the depot, traverses a set of arcs and ends its route at the depot. Each route cannot exceed a maximum time duration. The goal is to choose a set of the profitable customers and to design the routes of the vehicles in such a way that the required and chosen profitable customers are served, the time duration constraints of the routes are satisfied and the total profit collected is maximized.

More formally, a directed graph $G = (V, A)$ is given, where $V = \{1, \dots, n\}$ is the set of vertices and A is the set of arcs. Vertex 1 is the depot, that is the starting and ending vertex of each route. A travel time c_a is associated with each arc $a \in A$. Some arcs represent customers. The set $A_R \subseteq A$ represents customers that have to be served, whereas $A_P \subseteq A$ represents the set of profitable customers. A nonnegative profit s_a is associated with each arc $a \in A_P$. A fleet of K vehicles is available. The route of each vehicle cannot exceed a maximum time duration $T_{\max}$. The profit of any profitable customer can be collected by one vehicle at most. The objective of the TOARP is to maximize the total profit collected. A mathematical programming formulation for the TOARP can be found in Archetti et al. (2014).

3. A matheuristic for the TOARP

In this section we present a matheuristic for the solution of the TOARP that we call MAT (MAtheuristic for TOARP).

In the following, we say that a profitable arc is *served* by a vehicle if the vehicle traverses the arc and collects the corresponding profit. In MAT, there is no distinction between required and profitable arcs. A very large profit is assigned to the arcs in A_R and all arcs in $A_P \cup A_R$ are considered, and called, profitable. Thus, we redefine A_P as $A_P \cup A_R$. Moreover, the set of arcs A is completed by inserting all arcs between every pair of profitable arcs, plus the depot. Thus, if there is not an arc which links the head of a profitable arc (or the depot) with the tail of another profitable arc (or the depot), then we insert it in A. The cost of the inserted arcs is equal to that of the shortest path.

Before describing the different components of MAT, we introduce some notation and definitions.

3.1. Notation and definitions

The profit $S(C)$ of a set of profitable arcs $C \subseteq A_P$ is the total profit $\sum_{a \in C} s_a$. We denote by L_r the set of profitable arcs served by route r and by A_r the set of all arcs traversed by route r. The profit $S(r)$ of a route r is defined as the total profit of the profitable arcs served by the route, i.e., $S(r) = S(L_r)$. The duration $T(r) = \sum_{a \in A_r} c_a$ of a route r is its total travel time. A route r is *feasible* if it starts and ends at the depot and $T(r) \le T_{\max}$.

For a set R of routes, $S(R) = \sum_{r \in R} S(r)$ is the total profit of the routes in R and $L(R) = \bigcup_{r \in R} L_r$ is the set of profitable arcs served by the routes in R.

A *solution* s is defined as a set of routes such that each profitable arc is served by exactly one route. A solution s is said to be *feasible* if each route in s is feasible. Although in a solution s any arc (profitable or not) may be traversed more than once, the profit of a profitable arc is collected exactly once. We denote by $R_P(s)$ the set of the K most profitable routes in s (or the set of all routes in s if they are less than or equal to K), and $R_N(s)$ the set of all remaining routes. The aim of the TOARP is to determine a feasible solution s that maximizes $S(R_P(s))$.

The profitable arcs in $L(R_N(s))$ do not belong to the K most profitable routes in s, but are organized in routes. The reason for keeping these arcs organized into routes is that it is much easier to have a new route with a high profit by inserting profitable arcs in one of the routes in $R_N(s)$ than to create a new route from scratch. Although this requires an additional effort with respect to keeping the K most profitable routes only, it turned out to be beneficial to the efficiency of the heuristic. This solution structure was used effectively in the solution of other routing problems with profits like UCARPP (see Archetti et al., 2010), the TOP (see Archetti, Hertz, & Speranza, 2007), the