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Multi-objective models and techniques for analysing the absolute capacity of railway networks

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ABSTRACT

Railway capacity determination and expansion are very important topics. In prior research, the competition between different entities such as train services and train types, on different network corridors however have been ignored, poorly modelled, or else assumed to be static. In response, a comprehensive set of multi-objective models have been formulated in this article to perform a trade-off analysis. These models determine the total absolute capacity of railway networks as the most equitable solution according to a clearly defined set of competing objectives. The models also perform a sensitivity analysis of capacity with respect to those competing objectives. The models have been extensively tested on a case study and their significant worth is shown. The models were solved using a variety of techniques however an adaptive E constraint method was shown to be most superior. In order to identify only the best solution, a Simulated Annealing meta-heuristic was implemented and tested. However a linearization technique based upon separable programming was also developed and shown to be superior in terms of solution quality but far less in terms of computational time.

1. Introduction

Railway capacity determination and capacity expansion are increasingly important topics as railways become more developed, sophisticated and have greater demands placed upon them in the future. This article considers those tasks because there are significant limitations and weaknesses in prior approaches. Also there are many opportunities for the development of more advanced, sophisticated and all-encompassing techniques. Railway capacity determination and expansion activities have been addressed in a variety of different ways in past research. Some approaches have been purely analytical, however the majority have been empirical or simulation based. Recent examples that are noteworthy are: Dicembre and Ricci (2011), Singh et al. (2012), Mussone and Wolfler (2013), Yaghini (2014), Shih, Dick, Sogin and Barkan (2014), Froidh, Sipila and Warg (2014), Di Giandomenico, Fantechi, Gnesi and Itria (2013), Goverde, Corman and D'ariano (2013).

This article however specifically builds upon the research in Burdett and Kozan (2006), Kozan and Burdett (2005), and Burdett (2015) which predominantly considers the analysis of absolute capacity. In Burdett and Kozan (2006) the concept of absolute capacity was first defined and the value of analysing it was discussed in detail. In essence absolute capacity is a measure of the total possible throughput of trains across all the corridors of a railway network. It is an ideal level that only occurs when critical sections of rail are continuously occupied and train "interaction effects" and "interference delays" that are resolved by proper train scheduling (Burdett & Kozan, 2009a, 2009b), are ignored. Hence it is a form of structural analysis. Although absolute capacity is an overestimation of real "operational" capacity, it is sufficiently accurate for high level planning purposes, and thus provides a robust metric for benchmarking. This concept is also used in this article and is denoted forthwith by $\mathbb{A}$ and a number of other variations involving different sub scripting alternatives. It should be noted that this article does not consider how to optimise the performance of the network. To optimise the networks performance, maximising the number of trains can be detrimental as interaction effects and delays can be increased which reduce capacity utilisation.

The highlight of Burdett and Kozan (2006) was an optimisation approach that identifies the absolute capacity of a railway network, subject to a number of imposed technical constraints, and for a wide range of defined "real life" operational conditions such as the proportional mix of trains and their direction of travel, the length of trains, dwell times, and the presence of crossing loops and intermediate signals. In Kozan and Burdett (2005), rail access charging methodologies were also proposed. In Burdett (2015) supplementary analytical techniques for measuring and planning capacity expansion activities were developed. The new features in that article were track duplications and track sub divisions. Those developments identify how railway networks can be best expanded, either immediately or over time.

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The models developed in Burdett and Kozan (2006) and Burdett (2015) are essential techniques for absolute capacity determination however they assume a specific mix of trains is defined. Hence as only a single value of capacity is identified, those capacity models can only be used to identify how the infrastructure can be used and whether it can support an intended future traffic load. In practice, railway networks must be analysed for different mixes of trains. The number of different mixes that could be analysed is vast. To understand what occurs more generally, a sensitivity analysis or some other type of approach is required. In response a multi-objective approach is proposed in this article. The significance of a multi-objective approach is also that a variety of competing capacity metrics can be incorporated. In contrast the original model considered only a single objective which was the total number of trains, with for example no emphasis or meaning given to trains or to train services of different type. As few if any railways operate with a single train or service type, and those trains are not of equivalent worth, the previous models were somewhat inadequate. There are many ways to regulate competition and a multi-objective approach is investigated in this article as the best way to perform a sensitivity analysis of railways.

On the topic of multi-objective optimisation, several articles have considered railway applications. Ghoseiri, Szidarovszky and Ashharpour (2004) developed a multi-criteria optimisation approach to schedule trains. The competing objectives were fuel consumption and total passenger time. The Pareto frontier was determined using the e-constraint method and then a "distance" based method was utilised to solve the multi-objective decision problem. Twenty one modest sized test cases were solved. Zou and Zhong (2005) developed a branch and bound approach to schedule trains on double tracks. As the objective was bi-criteria, dominance rules were developed and Pareto solutions were generated. Lu et al. (2013) considered the development of a framework for evaluating the performance of railway networks. They proposed a "quality of service" framework that includes attributes like punctuality, resilience, energy/resource usage, journey time, etc. The proposed framework was considered because railway network performance is so multi-faceted. This affects both strategic and tactical planning. This article gives evidence (and motivation) of the need for more multi-objective planning approaches.

The articles by Kim and De Weck (2005, 2006) are generic and do not consider topics in railways. Those papers are however noteworthy for the development of multi-objective optimisation techniques. In their 2005 paper a method was developed for determining the Pareto front for bi-objective optimisation problems. Their approach is an adaptive version of the WSM, and is labelled AWS. It approximates a Pareto front by gradually increasing the number of solutions on the front. It concentrates computational effort where it is needed. In the 2006 paper multi-objectives were considered. A mesh of Pareto front patches was identified and refined.

2. Multi-objective capacity models

Access to railway networks is competitive in practice. As previously stated, railways are rarely constructed for specific services, and typically must sustain and support the movement of a mix of both passenger and freight services. There are a variety of different competitions that may be characterised and how this competition is regulated, greatly affects the outcome of any analysis of capacity. The following competition types are introduced: *service_versus_service*, *train_versus_train*, *corridor_versus_corridor*. These types all refer to competitions between trains. In summary trains of different service types (like passenger and freight) compete, but individual trains also compete against other trains. Similarly trains with different routes (i.e. that traverse different corridors) compete against those with other routes.

In Burdett and Kozan (2006) competition was regulated by the introduction of three parameters, namely the proportional mix of train types (i.e. the proportional distribution), the directional mix (i.e. the directional distribution), and the proportional flow of trains on different corridors (i.e. the percentage flow). These had to be specified a-priori, and were assumed to be static. These three parameters describe most of the different types of competition.

In this article several different multi-objective models have been developed to address the different forms of competition on the network. A base (i.e. core) capacity model however is first reviewed as the new models are based upon it.

2.1. The base capacity model

A review of the base model is presented here. In that model the set of train types is denoted by I, the set of corridors by C, and the set of section by S. The number of tracks on each section is denoted by τ_s and the set of sections in each corridor is denoted by Ω_c. The purpose of the model is to determine the maximum number of trains that can traverse the network over time T. Of the total number of trains selected by the model, the number of trains of each type is also identified. For example $\overrightarrow{x}_i^c$ and $\overleftarrow{x}_i^c$ are the number of trains of type i that traverse corridor c in each direction respectively. Similarly, $\vec{y}_i^s$ and $\overleftarrow{y}_i^s$ are the number of trains of type i that traverse each section s. The sectional occupation times in each direction are denoted by $\vec{T}_i^s$ and $\overleftarrow{T}_i^s$ and these are assumed to be provided. These values primarily include the sectional running time (SRT_i^s), but may also include other time spent on sections, such as loading or unloading passengers and freight, breakdowns, delays, acceleration, deceleration, etc. The SRT may be measured or theoretical values may be used. For example the free flow sectional running time is as follows: $\mathrm{SRT}_i^s = 60 \times L_s/V_i$ where L_s is the section length, V_i is the train speed, and 60 is multiplier needed to convert to minutes.

The base model has a set of standard constraints that regulate the mix of trains and the flow of trains across every section of the network over time. The mix regulation constraints are optional, and can be added or removed as desired. They are as follows:

$$\sum_{i\in I}\left(\overrightarrow{x}_i^c + \overleftarrow{x}_i^c\right) = \sigma_c \sum_{c'\in C}\sum_{i\in I}\left(\overrightarrow{x}_i^{c'} + \overleftarrow{x}_i^{c'}\right)\ \forall c \in C \qquad \text{[Mix across corridors]} \quad (1)$$

$$\overrightarrow{x}_i^c + \overleftarrow{x}_i^c = \eta_{c,i}\sum_{i'\in I}\left(\overrightarrow{x}_{i'}^c + \overleftarrow{x}_{i'}^c\right)\ \forall i \in I, c \in C \qquad \text{[Proportional mix using } \eta_{c,i}\text{]} \quad (2)$$

$$\overrightarrow{x}_i^c = \mu_{c,i}\left(\overrightarrow{x}_i^c + \overleftarrow{x}_i^c\right)\ \forall i \in I, c \in C \qquad \text{[Directional mix]} \quad (3)$$

The proportion of trains of each type in the forward direction is denoted by $\mu_{c,i}$ or by $\overrightarrow{\mu}_{c,i}$. Hence, $\overrightarrow{\mu}_{c,i} + \overleftarrow{\mu}_{c,i} = 1$. The flow of trains on each corridor proportional to that across all corridors is denoted as the percentage flow and is denoted by σ_c. The proportion of trains of each type on each corridor is denoted by $\eta_{c,i}$. Its definition regulates the competition between train types on different corridors. In other words on each corridor, the mix of trains is selected separately such that $\sum_{i\in I}\eta_{c,i} = 1 \quad \forall c \in C$.

An alternative is proposed here whereby $\eta_{c,i}$ could be redefined as the proportion of trains on each corridor of each type. In other words, for each train type, the mix is specified differently between corridors and such that $\sum_{c\in C}\eta_{c,i} = 1 \quad \forall i \in I$. To avoid confusion a separate parameter is defined for this situation, namely $\kappa_{c,i}$. Hence