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Decision Support

Undesirable outputs and weighting schemes in composite indicators based on data envelopment analysis

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ABSTRACT

This paper discusses different models that can be used to construct composite indicators with both desirable and undesirable output indicators. Two approaches are considered. The first is an indirect approach, based on a traditional Data Envelopment Analysis model, requiring a prior transformation in the measurement scale of the undesirable outputs. The second is a direct approach, based on a directional distance function model. The use of a directional distance function allows for the accommodation of undesirable indicators in their original form. The main limitations of these approaches are discussed related to the data transformation in the case of the indirect approach and the possibility to obtain negative margin rates of substitution between the desirable and undesirable outputs in the case of the direct approach. These issues lead to the proposal of a new composite indicator model based on a directional distance function that overcomes the limitations associated with the existing approaches. The incorporation of information on the relative importance of individual indicators using weight restrictions is discussed. Proposed here is an enhanced formulation of weight restrictions, in the form of assurance regions type I, that reflects the relative importance of the indicators in percentage terms. The models are illustrated in the assessment of Brazilian hydropower plants and are suitable for any assessment involving the aggregation of key performance indicators whenever undesirable outputs are present.

1. Introduction

The Data Envelopment Analysis (DEA) technique uses linear programming to evaluate the relative efficiency of a homogeneous set of Decision Making Units (DMUs) in their use of multiple inputs to produce multiple outputs. In standard DEA models, an inefficient DMU can improve its performance by increasing the levels of outputs (results obtained) or decreasing the levels of inputs (resources used). However, real world applications may involve both desirable and undesirable outputs and inputs. For example, in environmental performance assessment, we may have an output indicator related to quality of the water, for which more output corresponds to better performance and another output indicator related to the levels of CO_2 emissions, for which less output corresponds to better performance. In this situation, an inefficient DMU should increase the quality of the water or decrease the levels of CO_2 emissions to improve performance.

The performance measurement literature has addressed the measurement of productive efficiency in the presence of undesirable outputs since the 1980s. One of the earliest studies addressing the incorporation of undesirable outputs in the assessment of production efficiency was developed by Pittman (1983). This study extended the multilateral productivity indicator proposed by Caves, Christensen, and Diewert (1982) to include measures of both desirable and undesirable outputs. The multilateral productivity indicator developed by Caves et al. (1982) required the specification of price data, but this information is often unavailable for undesirable outputs. Therefore, Pittman (1983) proposed an extension of this indicator that assigned a value to the undesirable outputs based on estimates of shadow prices instead of market prices. Some years later, Fare, Grosskopf, Lovell, and Yaisawarng (1993) proposed an alternative method to estimate shadow prices based on the distance function defined by Shephard (1970). The specification of the shadow prices of undesirable outputs using a linear programming model allowed enhancing the approach proposed by Pittman (1983).

Fare, Grosskopf, Lovell, and Pasurka (1989) also proposed a modification in the Farrell's (1957) approach to efficiency measurement to allow an asymmetric treatment of desirable and undesirable outputs. While the multilateral productivity indicator requires the specification of the price information for the undesirable outputs, the

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nonparametric approach of Fare et al. (1989) only requires data on quantities of the undesirable outputs. The authors proposed a hyperbolic approach to efficiency measurement to allow considering different assumptions on the disposability of undesirable outputs. The new constraints state that the desirable outputs are strongly disposable (i.e. they can be reduced without cost), while the undesirable outputs are weakly disposable (i.e. they can be reduced only in conjunction with a reduction in the other outputs or an increase in the use of inputs).

Some years later, Chung, Fare, and Grosskopf (1997) introduced a different approach to deal with undesirable outputs in the efficiency and productivity measurement literature. The authors proposed the use of a directional distance function to allow expanding the desirable outputs while simultaneously contracting the undesirable ones. The directional distance functions has been widely used in the context of environmental performance assessment, in which the production of waste is often present.

The approaches mentioned above are known as direct approaches to treat undesirable outputs. These approaches allow treating the outputs in their original form, that is, without requiring any modification to the measurement scale. On the other hand, there are indirect approaches that transform the values of the undesirable outputs to allow treating them as normal outputs in traditional DEA models.

Scheel (2001), Dyson et al. (2001) and Seiford and Zhu (2002) discussed the different approaches for handling undesirable outputs in DEA models using indirect approaches. One option is to move the variables from the output to the input side. Scheel (2001) pointed out that this approach results in the same technology set as incorporating the undesirable outputs as normal outputs, in the form of their additive inverses ($-y_{\text{und}}$). Regarding this option, Seiford and Zhu (2002) pointed out that to treat undesirable outputs as inputs would not reflect the real production process, as the input–output structure that defines the production process would be lost. The incorporation of the undesirable outputs in the form of their additive inverses was first suggested by Koopmans (1951). Another possibility is to consider the undesirable outputs in the form of their multiplicative inverses ($1/y_{\text{und}}$), as proposed by Golany and Roll (1989). Regarding this option, Dyson et al. (2001) pointed out that this transformation would destroy the ratio or interval scale of the data. The third option is to add to the additive inverses of the undesirable outputs a sufficiently large positive number ($-y_{\text{und}} + M$), as suggested by Seiford and Zhu (2002). This transformation is the most frequently used in the literature to deal with undesirable outputs using a traditional DEA formulation (Cook & Green, 2005; Oggioni, Riccardi, & Toninelli, 2011).

In addition to the above mentioned approaches, in Cherchye, Moesen, Rogge, and Van Puyenbroeck (2011) the transformation in the measurement scale of the undesirable outputs was performed based on a normalization procedure, which was applied both to desirable and undesirable outputs. This procedure results in indicators varying between 0 and 1. As data normalization leads to a loss of information, this approach is rarely used in DEA studies. It does not take advantage of the ability of DEA to deal with data measured on different scales.

Although the papers mentioned above approach the presence of undesirable outputs and inputs in DEA models, they do not address the modeling of undesirable factors in the construction of composite indicators (CIs). A CI is given by the aggregation of several individual indicators in a single measure. CIs are intended to reflect multidimensional concepts that cannot be captured by a single indicator and they have benefits such as the capacity to summarize information, the facility to interpret results compared with a battery of separate indicators, and the capacity to reduce the visible size of a set of indicators without dropping the underlying base information (Nardo et al., 2008). Examples of well established composite indicators are the Environmental Performance Index (Emerson et al., 2012), Climate Change Performance Index (Burck, Hermwille, & Krings, 2012), and the Human Development Index (United Nations, 2013). They have been extensively used by decision makers to guide the definition of better policies to improve country performance.

The Organisation for Economic Co-operation and Development (OECD) and the European Commission provide a handbook for the construction of composite indicators that discusses the range of methodological approaches available to construct CIs (Nardo et al., 2008). The handbook highlights the growing interest in composite indicators by academic circles, the media, and policymakers. One point discussed and recognized as a source of contention is the definition of the relative importance of the indicators. The handbook indicates that DEA is an interesting weighting and aggregation procedure to reduce the inherent subjectivity associated with the specification of weights. As the indicator weights result from an optimizing process based on linear programming, they are less prone to subjectivity and controversy.

The use of DEA for performance assessments focusing only on achievements, rather than the conversion of inputs to outputs, was first proposed by Cook and Kress (1990), with the purpose to construct a preference voting model (for aggregating votes in a preferential ballot). Other relevant studies that support the empirical use of DEA models only with output indicators (or productivity indicators that aggregate output and input information, such as revenue per employee and GDP per capita) can be found in different fields, such as macroeconomic performance assessment (Lovell, Pastor, & Turner, 1995), selective examinations for university entrance (Hashimoto, 1996), university quality assessment (Murias, Miguel, & Rodriguez, 2007), human development (Despotis, 2004; 2005; Mahlberg & Obersteiner, 2001), technology achievement (Cherchye et al., 2008) and evaluation of urban quality of life (Morais & Camanho, 2011). In these studies, all variables were specified as outputs and an identical input level, which for simplicity was assumed to be equal to one, was specified for all DMUs.

In this paper, we approach two main issues: the construction of CIs that include both desirable and undesirable output indicators with aggregation procedures based on DEA, and the use of weight restrictions in this context. Two approaches are considered to construct the CI in the presence of undesirable indicators. The first is an indirect approach, based on a traditional DEA model, in which the undesirable output indicators require a prior transformation in the measurement scale to be accommodated in the CI model. The second is a direct approach in which the CI model is specified using a directional distance function. The use of a directional distance function allows for the accommodation of the undesirable output indicators in the CI model in their original form. The strengths and weaknesses of these approaches are discussed, leading to the proposal of a new CI model, also based on a directional distance function, that is able to overcome some limitations associated with the existing approaches. Concerning the use of weight restrictions in the context of estimation of CIs, the paper discusses the implementation of the two most popular types of weight restrictions: the virtual restrictions on weights and the assurance regions type I (ARI) weight restrictions. We propose an enhanced formulation of weight restrictions to incorporate the relative importance of individual indicators expressed in percentage terms, using ARI. The features, weaknesses and advantages of the alternative models and weight restrictions are discussed using a small example, which allows a graphical illustration of the results. Finally, a real world example consisting of the assessment of Brazilian hydropower plants is presented.

From a methodological perspective, the major contributions of this paper consist of the construction of DEA-based CIs that can accommodate both desirable and undesirable output indicators as well as the specification of a novel type of weight restriction, using ARI, to incorporate the relative importance of indicators, expressed in percentage terms.

The remainder of this paper proceeds as follows. Section 2 presents the DEA formulations that can be used for efficiency assessments in