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Degradation-based maintenance decision using stochastic filtering for systems under imperfect maintenance

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ABSTRACT

The notion of imperfect maintenance has spawned a large body of literature, and many imperfect maintenance models have been developed. However, there is very little work on developing suitable imperfect maintenance models for systems outfitted with sensors. Motivated by the practical need of such imperfect maintenance models, the broad objective of this paper is to propose an imperfect maintenance model that is applicable to systems whose sensor information can be modeled by stochastic processes. The proposed imperfect maintenance model is founded on the intuition that maintenance actions will change the rate of deterioration of a system, and that each maintenance action should have a different degree of impact on the rate of deterioration. The corresponding parameter-estimation problem can be divided into two parts: the estimation of fixed model parameters and the estimation of the impact of each maintenance action on the rate of deterioration. The quasi-Monte Carlo method is utilized for estimating fixed model parameters, and the filtering technique is utilized for dynamically estimating the impact from each maintenance action. The competence and robustness of the developed methods are evidenced via simulated data, and the utility of the proposed imperfect maintenance model is revealed via a real data set.

1. Introduction

Maintenance actions can be classified, according to their efficiency, into three types: perfect maintenance, imperfect maintenance and minimal maintenance. The assumption of perfect maintenance is only applicable to structurally simple systems, and the assumption of minimal maintenance is only applicable to highly complex systems. By contrast, it is more realistic in true experience that maintenance actions are imperfect, merely restoring a maintained system's condition to somewhere between good-as-new and bad-as-old. To evaluate the impact of maintenance actions on a maintained system's condition, many imperfect maintenance models have been developed. One commonly used class of imperfect maintenance models is the virtual age model introduced by Kijima (1989). The virtual age model assumes that an imperfect repair reduces a maintained system's physical age (therein called virtual age) by an amount proportional to the physical age just before the maintenance, or by an amount proportional to the additional age accumulated since the last maintenance. Many researchers have used the concept of virtual age for modelling imperfect corrective maintenance and/or imperfect preventive maintenance; see, among others, Doyen and Gaudoin (2011), Bouguerra, Chelbi, and Rezg (2012), Dijoux and Idee (2013), Ramirez and Utne (2013), Ahmadi (2014) and Ramirez and Utne (2015). Another commonly used class of imperfect maintenance models is the improvement factor model introduced by Malik (1979). The improvement factor model assumes that an imperfect repair changes the time of the hazard rate curve to some newer time but not all the way to zero. Lin, Zuo, and Yam (2000) extended the improvement factor model, stating that an imperfect repair could change both the time and the slope of the hazard rate curve. The extended improvement factor model hence includes many documented imperfect maintenance models as special cases, e.g., the virtual age model. For recent research on the (extended) improvement factor model, the reader may refer to, e.g., Park, Chang, and Lie (2012), Xia, Xi, Zhou, and Du (2012) and Khatab, Ait-Kadi, and Rezg (2014). Other imperfect maintenance models that cannot be classified into the improvement factor model are the Brown–Proschan model (see, e.g., Doyen, 2011) and the geometric process model (see, e.g., Zhang, Xie, & Gaudoin, 2013). Reviewing works on imperfect maintenance models are given, e.g., by Wang and Pham (2006), Nakagawa (2006) and Shafiee and Chukova (2013).

We note that most of the documented work on imperfect maintenance is only concerned with age-based maintenance. When dealing with degradation-based maintenance, people typically adopt the assumption that maintenance actions are either minimal or perfect

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(see, e.g., Chen, Ye, Xiang, & Zhang, 2015). The issue of treating imperfect maintenance in the context of degradation-based maintenance has not received much attention and remains widely open. Zhou, Xi, and Lee (2007) employed the improvement factor model for modeling the impact of imperfect repairs on a continuously monitored deteriorating system. However, one of the major shortcomings of the improvement factor model is that it is only applicable to cases where the hazard rate function can be derived analytically. For many models, e.g. the non-stationary Wiener process, no analytical hazard rate function is available. Wang and Pham (2011) treated imperfect maintenance by lifting the degradation critical threshold proportionally. Apparently, their approach is confined to the realm of speculation. It is more desirable to develop an imperfect maintenance model by taking a physically meaningful approach. Instead of lifting the degradation critical threshold, some researchers, e.g. Nicolai, Frenk, and Dekker (2009), Van and Berenguer (2012) and Mercier and Castro (2013), treated imperfect maintenance by reducing the degradation level of a maintained system by a random amount. The uniform distribution is always employed for modeling the distribution of the random amount. We should point out that the degradation level of a maintained system before and after a repair can be exactly known (e.g., by virtue of sensors). Therefore, the amount of degradation reduced by the repair can be exactly known. Hence, the practicability of the random degradation-level-reduction paradigm is debatable. What maintenance technicians need is a practical and useful imperfect maintenance model for degradation-based maintenance. Motivated by the need of maintenance technicians and inspired by the extended improvement factor model, the broad objective of this paper is to propose an imperfect maintenance model, called random improvement factor model, for degradation-based maintenance.

The random improvement factor model is founded on the intuition that maintenance actions will change the rate of degradation of a system, other than the level of degradation. The degradation rate defined herein is the rate of accumulation of degradation (see, e.g., Meeker & Escobar, 1998). Repairs slowing down degradation rate can be easily visualized in the context of, e.g., coating operations. In order to slow down the deteriorating process, steel structures are usually protected by certain organic coating systems (Perrin, Merlatti, Aragon, & Margaillan, 2009). The degradation level after each coating operation may or may not be changed. Another type of maintenance is lubricating rotating gears, by which the wearing processes of the gears will slow down. The degradation level after lubrication remains unchanged. One distinguishing feature of the random improvement factor model is the introduction of a latent variable, taking into account the fact that each maintenance action should have a different degree of impact on the rate of degradation. Accordingly, the filtering technique is utilized for dynamically estimating each realization of the latent variable. For illustrative purpose, the Wiener process is employed due to its adaptability in modeling degradation phenomena. The Wiener process (in its many forms) has been applied in Meeker and Escobar (1998) to model the size of fatigue crack as a function of the number of cycles, in Wang (2010) to model the degradation of bridge beams due to chloride ion ingression, and in Ye, Wang, Tsui, and Pecht (2013) to model the wear of magnetic heads used in hard disk drives and the light output of a light emitting diode. Applications of the Wiener process are also reported in Nikulin, Limnios, Balakrishnan, Kahle, and Huber-Carol (2010), Bian and Gebraeel (2012), Son, Fouladirad, Barros, Levrat, and Iung (2013), Si, Chen, Wang, Hu, and Zhou (2013) and Si, Wang, Hu, and Zhou (2014), to name a few. We shall underscore that, after appropriate modifications, the following procedure can be readily applied to other stochastic processes.

The remainder of the paper is organized as follows. Section 2 is devoted to presenting the random improvement factor model (in Section 2.1) and a generic maintenance scheme (in Section 2.2). Section 3 is devoted to real-time updating the degradation rate function of the maintained system (in Section 3.1) and the estimates of other fixed model parameters (in Section 3.2). Section 4 gives numerical examples to show the applicability and competence of the advanced techniques. Section 5 outlines concluding remarks and future work.

2. Model formulation

2.1. Random improvement factor model

Let $v(t)$ be a differentiable, non-negative, real-valued function of $t(\geq 0)$ with $v(0) = 0$. A Wiener process, denoted by $\{X_t, t \geq 0\}$, with drift function $v(t)$ and variance coefficient $\sigma(>0)$ is defined by $X_t = v(t) + \sigma W_t$; see, e.g., Si, Wang, Hu, Zhou, and Pecht (2012), Bian and Gebraeel (2012) and Son et al. (2013). $\{W_t, t \geq 0\}$ is the standard Brownian motion. The Wiener process has independent and normally distributed random increments. That is, for all $0 \leq s < t$, $X_t - X_s$ is independent of X_s and has the normal distribution $N(v(t) - v(s), \sigma^2(t-s))$. If $v(t)$ is a linear function, then $\{X_t, t \geq 0\}$ is called a stationary Wiener process. If $v(t)$ is a non-linear function, then $\{X_t, t \geq 0\}$ is called a non-stationary Wiener process. Suppose that a system, whose degradation conforms to the Wiener process $\{X_t, t \geq 0\}$, is put into operation at time 0. The expected degradation level of the system up to any time t is $\mathrm{E}[X_t] = v(t)$. Therefore, the first-order derivative of $v(t)$, denoted by $v'(t)$, can be treated like the degradation rate function of the underlying degradation process. After being put into operation at time 0, the system is repaired at time $s(>0)$. Right before the repair the degradation rate is $v'(s)$. Assume that the repair takes negligible time. The random improvement factor model says that right after the repair the degradation rate will be $bv'(s)$ with $0 < b < 1$. Here, b is a degradation-rate-reduction factor. It is noteworthy that each repair should have a different degree of impact on the degradation rate function. Therefore, the random improvement factor model says that the degradation-rate-reduction factor, b, is a random variable having certain distribution. Moreover, b is indeed a latent variable because the effect of a repair can only be inferred. From time s forward, the degradation rate function will be re-written by $bv'(s+t)$ for $t > 0$. Accordingly, from time s forward, the drift function will be re-written by $bv(s+t) + c$ for $t > 0$. Mathematically, the drift function is an integral of the degradation rate function. Hence, c can be treated like the constant of integration. Practically, c is introduced due to the fact that the repair may or may not shift the degradation level. Hence, we might call c a shifting factor.

Remark 1. Like the degradation-rate-reduction factor, the shifting factor should differ for different repairs. Though c is not fixed, it can be uniquely derived, given that all the other parameters are known (estimated). Suppose that a repair is performed at time $t_1(>0)$. The degradation level right before the repair is $x_{t_1^-} = v(t_1) + \sigma w_{t_1}$. w_{t_1} is a realization of W_{t_1}. The degradation level right after the repair is $x_{t_1^+} = b_1 v(t_1) + c_1 + \sigma w_{t_1}$. b_1 and c_1 are respectively the degradation-rate-reduction factor and shifting factor produced by the repair. The degradation measurement $x_{t_1^+}$ can equal or differ from the degradation measurement $x_{t_1^-}$, depending on the value of c_1. Once $x_{t_1^-}$ and $x_{t_1^+}$ are obtained, the shifting factor c_1 can be uniquely derived: $c_1 = x_{t_1^+} - x_{t_1^-} + v(t_1) - b_1 v(t_1)$. Therefore, every shifting factor can be evaluated till the end, when all the other unknown parameters (e.g., b_1) are estimated. Hence, in Section 3, we only developed methods for evaluating the other model parameters.

Remark 2. From time t_1 forward, the underlying degradation process can be modeled by $\{X_t = b_1 v(t) - b_1 v(t_1) + x_{t_1^+} + \sigma W_{t-t_1}, t > t_1\}$ which involves no shifting factor. Suppose that another repair is performed at time $t_2(> t_1)$. The degradation level at time t_2, right before the second repair, is $x_{t_2^-} = b_1 v(t_2) - b_1 v(t_1) + x_{t_1^+} + \sigma w_{t_2-t_1}$. $w_{t_2-t_1}$ is a realization of $W_{t_2-t_1}$. The second degradation increment is