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Discrete Optimization

A semidefinite optimization-based approach for global optimization of multi-row facility layout

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ABSTRACT

This paper is concerned with the Multi-Row Facility Layout Problem. Given a set of rectangular departments, a fixed number of rows, and weights for each pair of departments, the problem consists of finding an assignment of departments to rows and the positions of the departments in each row so that the total weighted sum of the center-to-center distances between all pairs of departments is minimized. We show how to extend our recent approach for the Space-Free Multi-Row Facility Layout Problem to general Multi-Row Facility Layout as well as some special cases thereof. To the best of our knowledge this is the first global optimization approach for multi-row layout that is applicable beyond the double-row case. A key aspect of our proposed approach is a model for multi-row layout that expresses the problem as a discrete optimization problem, and thus makes it possible to exploit the underlying combinatorial structure. In particular we can explicitly control the number and size of the spaces between departments. We construct a semidefinite relaxation of the discrete optimization formulation and present computational results showing that the proposed approach gives promising results for several variants of multi-row layout problems on a variety of benchmark instances.

1. Introduction

The general facility layout design problem is concerned with placing departments of given areas within a given facility. To each possible placement is assigned a cost based on the interactions between each pair of departments. These costs reflect an appropriate measure of adjacency preferences between departments. In some versions of the problem, the dimensions of the departments are also given. When this is not the case, finding their optimal shape is also a part of the problem. This problem is known to be NP-hard in general.

Versions of the facility layout problem occur in many practical contexts, not only in the planning of production and logistics facilities but also in applications such as VLSI chip design. A thorough survey of the facility layout problem is given in Meller and Gau (1996), where the papers on facility layout are divided into three broad areas. The first is concerned with algorithms for tackling the layout problem as defined above. The second area is concerned with extensions of the problem in order to account for additional issues that arise in applications, such as designing dynamic layouts by taking time-dependency issues into account, designing layouts under uncertainty conditions, and achieving layouts that optimize two or more objectives simultaneously. The third area is concerned with specially structured instances of the problem, such as the linear layout of machines along production lines. This paper is concerned with one such structured instance, namely the Multi-Row Facility Layout Problem (MRFLP).

An instance of the Multi-Row Facility Layout Problem (MRFLP) consists of a set of rectangular departments, a given number of rows, and weights for each pair of departments. We assume without loss of generality that each department can be assigned to any of the given rows, that the rows all have the same height, that the distances between adjacent rows are all equal, and that the departments all have the same height (equal to the row height). The problem is to find an assignment of departments to rows and the positions of the departments in each row so that the total weighted sum of the center-to-center distances between all pairs of departments is minimized. The (MRFLP) has many applications such as computer backboard wiring (Steinberg, 1961), campus planning (Dickey and Hopkins, 1972), scheduling (Geoffrion & Graves, 1976), typewriter keyboard design (Pollatschek, Gershoni, & Radday, 1976), hospital layout (Elshafei, 1977), the layout of machines in an automated manufacturing system (Heragu and Kusiak, 1991), balancing hydraulic turbine runners (Laporte and Mercure, 1988), numerical analysis (Brusco and Stahl, 2000) and optimal digital signal processors memory layout generation (Wess and Zeitlhofer, 2004). There has not been much research done on metaheuristic approaches

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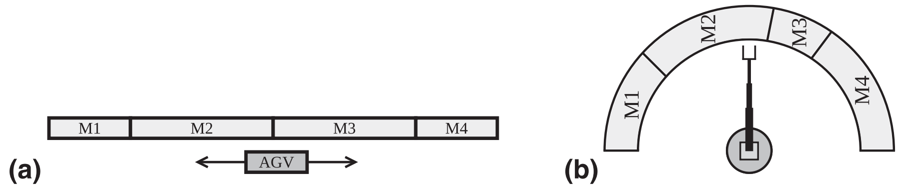

Fig. 1. In (a) an AGV transports parts between the machines moving in both directions along a straight line. In (b) a material-handling industrial robot carries parts between the machines.

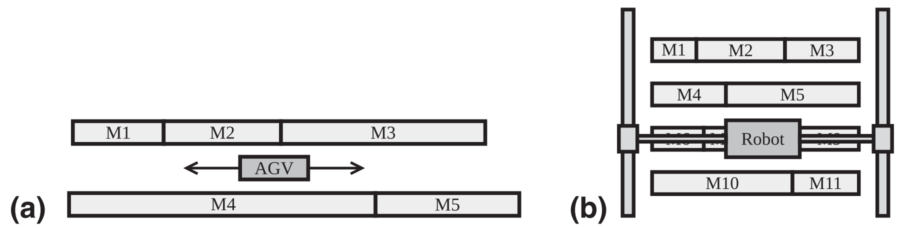

Fig. 2. In (a) an AGV transports parts between the machines that are located on both sides of a linear path of travel. In (b) a gantry robot is used when the space is limited.

to row layout problems with spacing, see e.g. the recent papers of Murray, Smith, and Zhang (2013) and Zuo, Murray, and Smith (2014). In this paper, we focus on mathematical programming approaches that can certify global optimality of solutions, or at least provide a guaranteed bound on the gap to optimality.

Row layout problems in general are of special interest for optimizing flexible manufacturing systems (FMSs). FMSs are automated production systems, typically consisting of numerically controlled machines and material handling devices under computer control, which are designed to produce a variety of parts. In FMSs the layout of the machines has a significant impact on the materials handling cost and time, on throughput, and on productivity of the facility. A poor layout may also negate some of the flexibilities of an FMS (Hassan, 1994). The type of material-handling devices used such as handling robots, automated guided vehicles (AGVs), and gantry robots typically determines machine layout in an FMS (Nearchou, 2006).

Possible row layout types are single-row (Fig. 1), double-row and multi-row layout (Fig. 2). The Single-Row Facility Layout Problem (SRFLP) requires that the departments be placed next to each other along a single row; this simplifies the problem significantly. In particular, there is no need to assign each department to a row, and the optimal solution will not have any empty space between departments. Therefore solving the (SRFLP) consists of finding the optimal permutation of the departments. This problem arises for example as the problem of ordering stations on a production line where the material flow is handled by an AGV travelling in both directions on a straight-line path (Heragu and Kusiak, 1988) (see Fig. 1). Several heuristic algorithms have been suggested to tackle large instances; the best ones to date are Datta, Amaral, and Figueira (2011), Kothari and Ghosh (2013a, 2013b, 2014) and Samarghandi and Eshghi (2010).

Three problems closely related to the (SRFLP) are the Single-Row Equidistant Facility Layout Problem (SREFLP), the Linear Arrangement Problem (LAP), and the *k*-Parallel Row Ordering Problem (kPROP). The (SREFLP) is the special case of the (SRFLP) with all departments equal in shape and the positions where they can be placed on the rows fixed in advance; it is sometimes also called one-dimensional machine location problem (Sarker, Wilhelm, & Hogg, 1998) or linear machine-cell location problem (Yu & Sarker, 2003). On the other hand, the (kPROP) is an extension of the (SRFLP) that considers arrangements of the departments along more than one row but with each department being assigned to a specific row in advance; hence the objective of the (kPROP) is to find a permutation of the departments within each row so that the total weighted sum of the center-to-center distances between all pairs of departments (with a common left origin) is minimized. If the (kPROP) is restricted to two rows we simply call it (PROP). Applications of the (kPROP) are the arrangement of departments along two or more parallel straight lines on a floor plan, the construction of multi-floor buildings, and the layout of machines in FMSs. We mention that even the (LAP), which is a special case of the (SREFLP) where all weights are binary, is already an NP-hard problem (Garey, Johnson, & Stockmeyer, 1974), and it remains so even if the underlying graph is bipartite (Garey & Johnson, 1979).

In layout problems, the pairwise connectivities are usually assumed to be non-negative to ensure boundedness of the objective value of the optimal layout. For the (SRFLP) and the other problems closely related to it, this further guarantees that all departments are placed next to each other without spacing. More generally, the (kPROP) can be further extended to the Space-Free Multi-Row Facility Layout Problem (SF-MRFLP) in which the optimization is also carried out with respect to the row assignments. This is a particular version of the (MRFLP) in which all the rows have a common left origin and no empty space is allowed between departments (Hungerländer & Anjos, 2012). If we restrict the (SF-MRFLP) to two rows we obtain the Space-Free Double-Row Facility Layout Problem (SF-DRFLP) as a special case that has been applied in spine layout design. Spine layouts, introduced by Tompkins (1980), require departments to be located along both sides of specified corridors along which all the traffic between departments takes place. Although in general some spacing is allowed, layouts with no spacing are much preferable since spacing often translates into higher construction costs for the facility.

The special case of the Double-Row Facility Layout Problem (DRFLP) can be viewed as a natural extension of the (SRFLP) in the manufacturing context when one considers that an AGV can support stations located on both sides of its linear path of travel (see Fig. 2). This is a common approach in practice for improved material handling and space usage. Furthermore, since real factory layouts most often reduce to double-row problems or a combination of single-row and double-row problems, the (DRFLP) and its space-free counterpart (SF-DRFLP), sometimes called the corridor allocation problem, are especially relevant for real-world applications.

Toy example. We use a small example to illustrate the differences between the (SRFLP), the (kPROP), the (SF-MRFLP) and the (MRFLP). We consider a problem with two rows so that we actually have instances of the (SRFLP), (PROP), (SF-DRFLP) and (DRFLP) respectively. Consider four departments with lengths $\ell_1 = 1,\ \ell_2 = 2,\ \ell_3 = 3,\ \ell_4 = 4$ and pairwise connectivities $c_{12} = c_{14} = c_{34} = 1,\ c_{13} = c_{24} = 2$. Fig. 3 illustrates the optimal layouts and the associated costs for the four different problems: