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Exact and heuristic linear-inflation policies for an inventory model with random yield and arbitrary lead times

K. Inderfurth, G. P. Kiesmüller*

Faculty of Economics and Management, Otto-von-Guericke-University Magdeburg, POB 4120, D-39016 Magdeburg, Germany

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ABSTRACT

We investigate a periodic-review inventory system for a single item with stochastic demand and random yield. Since the optimal policy for such a system is complicated, we study the class of stationary linear-inflation policies where orders are only placed if the inventory position is below a critical stock level, and where the order quantity is controlled by a yield inflation factor. We consider two different models for the uncertain supply, namely binomial and stochastically proportional yield, and we allow positive and constant lead times as well as asymmetric demand and yield distributions. In this paper we propose two novel approaches to derive optimal and near-optimal numerical values for the critical stock level, minimizing the average holding and backorder cost for a given inflation factor. First, we present a Markov chain approach which is exact in case of negligible lead time. Second, we provide a steady-state analysis to derive approximate closed-form expressions for the optimal critical stock level. We conduct an extensive numerical study to test the performance of our approaches. The numerical experiments reveal an excellent performance of both approaches. Since our derived formulas are easily implementable and highly accurate they are very valuable for practical application.

1. Introduction

It is well-known that in many industries production and inventory systems simultaneously have to cope with uncertainties from two sides, namely from the demand as well as from process side. A major risk concerning production processes stems from yield uncertainties, which is common in the agricultural sector or in chemical, electronics and mechanical manufacturing industries (see Gurnani, Akella, & Lehoczky, 2000; Jones, Lowe, Traub, & Kegler, 2001; Kazaz, 2004). In these environments different reasons such as weather conditions, unreliable processes or imperfect input material can be responsible for the occurrence of random yields. A particularly striking example is given by semiconductor manufacturing in the electronic goods industry where yield losses can be as high as 80 percent (see Nahmias, 2009, p. 392). The most recent field where yield problems have gained specific attention is the remanufacturing industry. Here, the output of disassembly operations often is highly uncertain because of limited knowledge of the condition of used products (see Ilgin & Gupta, 2010; Panagiotidou, Nenes, & Zikopoulos, 2013).

In coping with the described yield problems, production and inventory control faces three main challenges. First, it has to be recognized that yield losses are often hard to predict so that their variability is too high to be ignored and randomness of production output must explicitly be taken into account. Second, it is important to be aware that different causes of yield losses exist so that different types of yield randomness have to be dealt with. Finally, many production processes in this context, e.g. in semiconductor manufacturing, are characterized by significant production lead time so that lead times of different length must be considered when control systems are developed and implemented. In this paper, we study efficient approaches for controlling periodic-review production and inventory systems with random demand and yield, which are easy to implement and address all major challenges mentioned above. We refer to an infinite-horizon problem for a single item under the criterion of average cost minimization. Concerning yield randomness, we consider two different types that are most widely addressed in literature (see Yano & Lee, 1995), namely the models of binomial and stochastically proportional yield. Binomial yield applies to cases where the generation of non-defective units within a production batch forms a Bernoulli process while the stochastically proportional yield concept is used if the stochastic process conditions affect the batch as a whole so that the probability distribution of the yield rate does not depend on the batch size.

From prior research it is known (see Gerchak, Vickson, & Parlar, 1988; Henig & Gerchak, 1990) that the optimal policy in case of zero lead time and stochastically proportional yield is a rule with a

* Corresponding author. Tel.: +49 391 67 58797; fax: +49 391 67 41168.
E-mail addresses: karl.inderfurth@ovgu.de (K. Inderfurth), gudrun.kiesmueller@ovgu.de (G.P. Kiesmüller).

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critical stock such that a production order in a period is only released if the inventory does not exceed this stock level. Different from the standard base-stock rule under deterministic yield conditions, however, the order quantity is a non-linear function of the inventory position. Since this rule is computationally cumbersome to evaluate and unattractive for practical application, several approaches have been proposed to develop simple, but still near-optimal rules. Some of these approaches, like the so-called newsvendor and NLH-2 heuristic by Bollapragada and Morton (1999) and the heuristic developed by Li, Xu, and Zheng (2008), try to approximate the optimal order quantity rule by a non-linear function that is quite simple to evaluate. More convenient for practical application, however, are those policies where the order release quantity is a linear function of the deviation between critical stock and inventory position. This type of policy is often called linear-inflation rule (see Zipkin, 2000, p. 393) and has the major benefit that it possesses just the structure of the rule that is usually applied in practice when demand and yield variability in MRP-type of production control is handled by a safety stock and a yield inflation factor that accounts for yield losses (see Inderfurth, 2009; Nahmias, 2009, p. 392; Vollmann, Berry, Whybark, & Jacobs, 2005, p. 485).

A linear-inflation policy is characterized by two parameters, which have to be determined before implementation, namely the critical stock S and the yield inflation factor F. Like other research contributions in this field we will focus on this policy type although we are aware that this rule is non-optimal, not only due to the linearization of the order release function but also because it is myopic and uses static policy parameters even in the non-zero lead time case (see Bollapragada & Morton, 1999 and Inderfurth & Vogelgesang, 2013, respectively). From numerical studies in Bollapragada and Morton (1999) and in Huh and Nagarajan (2010), we know that the latter aspects are of minor relevance for suboptimality and that the performance of the linear-inflation policy critically depends on the choice of the numerical values of the policy parameters S and F.

Up to now, in literature we find four approaches, listed in Table 1, that aim to determine one or both of these policy parameters such that they are optimal or close to optimal. These approaches differ with respect to several problem aspects and application fields which are relevant in the context under consideration. First, these differences relate to the question which of the two policy parameters is determined and if this parameter evaluation is exact or heuristic. Further, the approaches can be divided into those which only consider zero lead time and others where non-zero lead times are also taken into account. A next criterion is if the approach only refers to a single yield model or if both yield types (binomial, BI, and stochastically proportional, SP) are considered. Finally, we find a difference in the types of probability distributions for demands and yields assumed in the approaches, namely if they are strictly symmetric or if asymmetric distributions are also allowed. Table 1 gives an overview how the four approaches refer to these aspects.

The first three approaches in Table 1 address a similar limited problem area as they all are restricted to a situation with zero lead time, stochastically proportional yield, and symmetric probability distributions. Under these conditions Bollapragada and Morton (1999) propose a heuristic approach (named NLH-1) resulting in a closed-form expression for the critical stock level S while they use the reciprocal of the expected yield rate as inflation factor F. Although there is a flaw in the calculations for this approach (see Inderfurth & Transchel, 2007), a numerical study reveals that this heuristic performs very well in most of the considered cases. The approach suggested by Zipkin (2000) presents a heuristic for determining closed-form solutions for both parameters S and F. Since these parameter formulas, however, are not derived on the basis of a strict cost minimization approach it turns out that, in general, the performance of the Zipkin heuristic is not satisfactory. This is shown in a computational study carried out by Huh and Nagarajan (2010) whose main own contribution is to develop a method for computing the optimal critical stock S for a given yield inflation factor F which is computationally tractable. Nevertheless, since for this approach a simulation procedure is needed to generate the stationary inventory distribution the computational burden is still quite considerable. Huh and Nagarajan (2010) also check different simple approaches for determining the inflation factor F and suggest a combined approach that works quite well compared to the optimal choice of this parameter. The fourth contribution in this sequence, published in Inderfurth and Vogelgesang (2013), extends the approach of S determination in Bollapragada and Morton (1999) and presents closed-form expressions for the critical stock also for binomial yield problems and for environments with non-zero lead times and asymmetric binomial yield rate distributions. For the non-zero lead time case, a numerical study reveals that the performance of the linear-inflation rule with the suggested static parameters is comparable to that of a more sophisticated rule with dynamic critical stock parameters which in each period specifically considers the yield risks from the respective open orders and, in principle, has the potential to perform better under non-zero lead times.

In our paper, we develop two novel approaches to derive optimal and near-optimal expressions for the critical stock S under arbitrary inflation factors F, which apply to all relevant problem instances, i.e. zero and non-zero lead times, binomial and stochastically proportional yield and arbitrary demand and yield distributions which also can incorporate considerable skewness. Both approaches aim to derive the probability distribution of the stationary inventory level under a linear-inflation policy and determine the optimal policy parameter from exploiting this distribution. The first approach uses a Markov chain modeling method and leads to a direct numerical computation of the inventory distribution which is exact for zero and unit lead time and is approximate for lead times greater than one. Thus, this Markov chain approach is able to determine the optimal critical stock level S like the Huh/Nagarajan approach, but with lower computational effort, because the cumbersome estimation of probabilities by stochastic simulation can be avoided and only a single linear equation system has to be solved. The second approach fits estimations of the moments of the stochastic stationary inventory to standard distribution types so that closed-form expressions for the critical stock parameter S can be found. For estimating the moments in this so-called steady-state approach, an approximate method is

Table 1
Approaches for parameter determination of the linear-inflation policy.

	Exact		Heuristic		Lead time		Yield		Distribution	
	S	F	S	F	Zero	Non-zero	SP	BI	Symmetry	Asymmetry
Bollapragada and Morton (1999)			√		√		√		√	
Zipkin (2000)			√	√	√		√		√	
Huh and Nagarajan (2010)	√			√	√		√		√	
Inderfurth and Vogelgesang (2013)			√		√	√	√	√	√	(√)[a]
This paper	√[b]		√		√	√	√	√	√	√

[a] Asymmetry is only considered for the yield distribution with restriction to binomial yield.
[b] Exactness is given for a lead time of zero and one period.