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Optimal ordering for a probabilistic one-time discount

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ABSTRACT

We develop a model for a one-time special purchasing opportunity where there is uncertainty with respect to the materialization of the discounted purchasing offer. Our model captures the phenomenon of an anticipated future event that may or may not lead to a discounted offer. We analyze the model and show that the optimal solution results from a tradeoff between preparing for the special offer and staying with the regular ordering policy. We quantify the tradeoff and find that the optimal solution is one of four intuitive policies. We present numerical illustrations that provide additional insights on the relationship between the different ordering policies.

1. Introduction

A basic premise in most inventory management models is that the unit-price is constant. Suppliers, however, are often under pressure to increase sales or reduce inventory levels. As a result, they tend to offer discounts so that retailers will be inclined to order larger quantities than usual. From a retailer's point-of-view, this may constitute an opportunity to benefit from higher sales margins.

For example, consider a retail chain selling a commodity, the behavior of which is relatively static. As the Olympics approach, the retailer identifies a special purchasing opportunity caused by potential overstocking of the commodity at suppliers operating in the vicinity of the hosting city. Such opportunity entails uncertainty, and its materialization is not guaranteed. The retailer faces a tradeoff between reducing stocks prior to the event to benefit from the discounted offer and risking higher inventory costs if the offer does not materialize. It is this tradeoff that we wish to resolve in our work.

The supplier also may not know for sure if the discount will be possible. Moreover, she may be reluctant to share information as it will obligate her. Thus, from the retailer's perspective, quantifying the potential savings that might be gained from preparing for a special purchasing opportunity can shed light on the effort and cost that it is worthwhile to invest in obtaining this information. In Lee, So, and Tang (2000), the authors show the impact the variability of demand has on the value of information. In our paper, we show that even in an otherwise deterministic setting, the possible occurrence of a one-time unit-price discount is influential.

We use the economic order quantity (EOQ) model as our modeling framework. Even though the model relies on a set of simple and restrictive assumptions, its simplicity and robustness (in the sense that a relatively large deviation in the estimation of the model's parameters results in a relatively small deviation in the objective function value) makes it a widely used tool.

We develop a model that depicts a system in the EOQ setting in which the retailer predicts that at a known point of time, a one-time special purchasing opportunity may occur with a certain probability. We analyze the optimal replenishment policy in such a setting, and characterize its structure. We then show that the optimal replenishment policy is one of four intuitive policies, which can be found and evaluated using closed-form expressions. Finally, we provide additional insights on the relation between the different policies through numerical illustrations.

The unit-price discount is extensively studied in the literature (see, e.g., Goyal, Srinivasan, & Arcelus, 1991 and Ramasesh, 2010). The setting we study in this research is unique in the sense that we tackle a one-time probabilistic discount, which can happen at a known time in the future. The next section provides a comprehensive review of this topic.

The rest of this work is organized as follows: After reviewing the literature in Section 2, we formulate our problem in Section 3. In Section 4 we analyze the model and derive an optimal solution. In Section 5 the results are visualized and studied using numerical illustrations. In Section 6 we discuss several extensions to our work. Finally, in Section 7, we highlight the main findings and suggest

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further research directions. To aid readability, the technical proofs are presented in Appendix A.

2. Literature review

The EOQ model is introduced by Harris (1913) and further developed by Wilson (1934). An elaborate review of the model and its well-known extensions can be found in Nahmias (2008). One of the first works on inventory models with discounts is that of Friend (1960) who study random purchasing opportunities with reduced fixed ordering costs. In their model, demand and discounted offers arrive as a Poisson processes, and the system is modeled as a Markov chain. Naddor (1966) introduce multiple discount models based on the EOQ model, including an immediate price increase. The author assumes that an immediate price increase occurs when the inventory level is zero, which prompts a one-time opportunity to purchase inventory at a discounted unit price. While the body of literature on unit-price discounts in inventory models is quite broad, Friend (1960) and Naddor (1966) exemplify the two main lines of work. The first, on recurrent discounts, usually operates in an infinite horizon setting, and the second, on one-time purchasing opportunity, often occurs in the EOQ environment. We begin with the latter, which is the focus of our work, and for completeness, we also note some of the salient studies on recurrent discounts.

One-time discounts appear in the literature mainly in one of three forms: (1) instantaneous discounts in the present or at a future time, (2) discounts over an interval of time, and (3) discounts in the form of an extended credit period. Ardalan (1988) studies a model that is similar to that of Naddor (1966) but allows an arbitrary level of inventory when the special opportunity takes place. Taylor and Bradley (1985) extend the model proposed by Naddor (1966) to account for an announced price increase at a future moment in time. Lev and Weiss (1990) consider the case of a finite horizon EOQ model with a single price change. They base their results on the work of Schwartz (1972), who investigate the EOQ model in a finite horizon setting. They use the total cost as the objective function. Tersine and Schwarzkopf (1989) study a model with an announced special purchasing opportunity for a nonrestrictive time duration in an infinite time horizon setting. Aull-Hyde (1996), Cardenas-Barron, Smith, and Goyal (2010), Al Kindi and Sarker (2011), and Taleizadeh, Pentico, Aryanezhad, and Ghoreyshi (2012) investigate a one-time discount when shortages are allowed. Several authors study similar problems assuming that costs are discounted (for example, see Aucamp & Kuzdrall, 1989; Grubbstrom & Kingsman, 2004).

Other types of unit-price discount models include the work of Davis and Gaither (1985) who study an EOQ model with a one-time offer of delayed payments. Ardalan (1994) looks at a one-time discount where the demand is sensitive to the discounted price. Arcelus, Shah, and Srinivasan (2003) look at different one-time price incentives for a model for perishable items. Other work on deteriorating or imperfect inventory include those by Chang, Lin, and Ho (2011), Allah Taleizadeh, Mohammadi, Eduardo Cardenas-Barron, and Samimi (2013), and Kevin Hsu and Yu (2009). An alternative discount model is introduced in Sari, Rusdiansyah, and Huang (2012), where there are several discount offers, with different discounts at different times that can be utilized at most once. Yang, Ouyang, Wu, and Yen (2012) study a one-time discount model where there is capacity constraint, while Peter Chu, Chen, and Thomas Niu (2003) look at a model where the discounted purchased quantity can be selected from a restricted set of values. Sarker and Al Kindi (2006) study multiple types of one-time discounts, including one that is both a one-time and quantity discounts. For an excellent review of one-time discount models, the reader is referred to Ramasesh (2010) and Goyal et al. (1991).

One challenge when measuring the performance of policies in an infinite horizon is that one-time disturbances in inventory behavior do not affect the long-run average cost. This means that the standard EOQ model objective function does not capture these disturbances. The most common approach to overcoming this difficulty is by optimizing the savings comparing to the EOQ model. This can be done in several ways, the most common being to consider the costs in a finite time interval until the system reverts to the EOQ settings, and approximating the EOQ costs during this interval. The conventional method of approximating the EOQ costs is to multiply the duration of the finite interval by the average cost the EOQ model. We denote this method as the Differential Costs approach (also known as the Average Cost Approach in Ramasesh, 2010). The need to approximate the EOQ costs comes from the fact that after the system reverts to the EOQ model, the difference in costs is not constant but rather cyclic and depends on the time when costs are compared. Approximating the EOQ costs, and looking at a finite interval resolves this problem. Two other methods for approximating the costs are offered by Yanasse (1990), who use the worst-case value of the cyclic difference, and Huang, Kulkarni, and Swaminathan (2003) and Lim and Rodrigues (2005), who consider the average difference over the cycle. An alternative approach is to consider discounted costs, a method that is known as the Discounted Cash Flow approach. This method results in a finite cost expression that can then be minimized; see also the discussion in Baker and Hanna (1987).

For recurrent discounts, Hurter and Kaminsky (1968b) extend the model presented by Friend (1960) to incorporate discounts on the unit price. Hurter and Kaminsky (1968a) study a different discount model where the system randomly transitions between two states, with high and low costs. They consider a policy that is a mixture of a base-stock policy, and an (s, S) policy for the high and low intervals, respectively. Kalymon (1971) study a periodic review inventory model in which the unit price changes according to a Markov chain, whereas in Golabi (1985), a periodic review model with random costs and fixed demand is analyzed. Silver, Robb, and Rahnama (1993) develop effective heuristics for the model presented by Hurter and Kaminsky (1968b). Zheng (1994) prove the optimality of a threshold-based policy for a continuous review inventory system with Poisson process demand, while Moinzadeh (1997) investigate an EOQ model with random deal offerings. Finally, we note the work by Chaouch (2007) that generalizes Hurter and Kaminsky (1968a) to allow for demand that is affected by cost, and also low- and high-price intervals of different lengths.

In this study, we generalize existing work by incorporating a probabilistic one-time discount offer. This generalization is usually overlooked by assuming that the discount offer occurs without notice. Our model captures an ubiquitous phenomenon with considerable economical impact that has not received much attention in the literature, where traditionally it is assumed that one-time discounts materialize with certainty. Moreover, our analysis offers a unified closed-form solution to the problem that is both insightful and intuitive, and can be easily implemented in practice.

3. Model

Our model is best thought of as the classical EOQ model with an additional assumption of a one-time special purchasing opportunity. The EOQ parameters are the demand rate D, fixed order cost K, unit cost c, and holding cost rate h. Shortages are not allowed. We add to the classical EOQ model a one-time special purchasing opportunity at a future time t_s that may materialize with a known probability p. Without loss of generality, we assume that inventory is zero at the beginning of the planning horizon. The case in which the initial inventory is greater than zero is discussed in Section 6.1. Table 1 summarizes our notation. We use $*$ to denote optimal values.

The classical EOQ objective function is the long-run average cost. Thus, any deviation from an ordering policy over a finite-time horizon will not affect the objective function value. This means that a