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Modeling the modal split and trip scheduling with commuters' uncertainty expectation

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ABSTRACT

This paper investigates the modal split and trip scheduling decisions with consideration of the commuters' uncertainty expectation in the morning commute problem. Two physically separated modes for transportation, the auto mode on highway and the transit mode on subway, are available for commuters to choose for traveling from home to workplace. The travel time uncertainty is assumed to only occur on highway. Every commuter is faced with the joint choice of transport mode and trip scheduling for minimizing her/his generalized travel cost. The study reveals that uncertainty expectation can significantly influence the travel decisions and lead to a distinctive flow pattern. We also examine the effects of transit headway and fare on modal split and equilibrium cost by numerical examples.

1. Introduction

Most cities are facing growing traffic demand and limited road space, which definitely gives strong reason to rapidly develop mass transit. In a network with multiple transportation modes, the home departure choice of commuters always involves two dimensions of decision making such as which mode to select and when to depart. It is evident that the study of jointly considering modal split and trip scheduling is important for us to obtain deep insights into how to manage congestion dynamically and allocate commuters among multiple modes efficiently. This has received great attention during last decades (Danielis & Marcucci, 2002; Huang, 2000; Kraus, 2003).

The pioneering model with endogenous trip scheduling was developed by Vickrey (1969). In his model, congestion takes the form of queuing behind a single bottleneck and each commuter is confronted with a trade-off between queuing cost and schedule delay cost of early or late arrival at workplace. This work has been extended to a variety of circumstances and widely studied (Arnott, de Palma, & Lindsey, 1990; van den Berg & Verhoef, 2011; Xiao, Qian, & Zhang, 2011). Lindsey (2004) presented an analysis of the existence and uniqueness of the user equilibrium with consideration of user heterogeneity. Zhang, Yang, Huang, and Zhang (2005) examined the linkage between morning and evening commutes by allocating the commuters' utility-valued work time. de Palma and Fosgerau (2013) generalized the bottleneck congestion analysis through allowing for random queue and considering the users' scheduling preferences.

The above mentioned studies only focused on the morning commute problems with single mode. A fact in reality is that there often exist multiple modes providing substitutable transportation services, e.g., commuters in most cities can make trips either by private cars or public transit. Hence, a topic of modal split arises and has received longstanding interests (Huang, Tian, Yang, & Gao, 2007; Huang & Yang, 1999; Wu, Yin, & Lawphongpanich, 2011). Tabuchi (1993) was among the first to study the modal split behavior in a transportation system with a physically separated mass transit parallel to a highway with a bottleneck. He compared the optimality and efficiency of several transit fare and road toll regimes. Huang (2000, 2002) extended Tabuchi's work by considering the user heterogeneity and elastic demand. Suggesting that the transit mode is represented by buses traveling on the same road with private cars, Huang et al. (2007) studied the mode choice and commuting behaviors in a newly proposed bi-modal transportation system. Mirabel and Reymond (2010) investigated the redistribution of toll revenue and analyzed the impact of redistribution on total cost and modal split. Habib (2012) formulated a combined model of mode choice, work start time and work duration, and revealed some behavioral insights into activity scheduling and mode choice decisions. Gonzales and Daganzo (2012) examined the two-mode problem with time-dependent demand in an urban area and optimized the transit fare for minimizing the total system cost. Tian, Yang, and Huang (2013) recently investigated the efficiency of a tradable travel credit scheme designed for a highway/transit system with user heterogeneity in value of time.

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For a competitive two-mode transportation system, there is an important extension valuable to conduct, i.e., the impact of highway travel time uncertainty on modal split and trip scheduling decisions. The importance has been recognized by many scholars (Brownstone & Small, 2005; Fosgerau & Karlstrom, 2010; Siu & Lo, 2008), and some progresses have been obtained. Noland and Small (1995) determined the optimal departure time under uniform and exponentially distributed random delay with an exogenous time profile of congestion. Noland and Polak (2002) reviewed both the theory and empirical results of several cases for measuring the coefficient of travel time variability using the stated preference techniques. Siu and Lo (2009) examined the role of uncertainty in affecting the equilibrium trip scheduling, using a model incorporating the user heterogeneity in arrival earliness and lateness.

It is commonly observed that driving on highway is easily influenced by external random incidents, while the transit runs are always operated in a regular manner with invariable travel time. Inspired by this observation, this study assumes that only the travel time on highway is uncertain and commuters are homogeneous in their own uncertainty expectation.[1] Different from what is proposed in Siu and Lo (2009), the exponential distribution for travel delay is employed, and thus the influence of uncertainty expectation on travel decisions can be carefully stated. According to their perceived uncertainty expectation, commuters consciously adjust the choices of transport mode and departure time to minimize the expected generalized travel cost. Without loss of generality, we deal with a two-mode transportation system in which the auto mode on highway and the transit mode on subway compete to serve a population of commuters. Commuters have to first make a trade-off between experiencing the uncertain risk of arriving early or late if selecting auto mode and bearing the body congestion occurring in carriages if selecting transit mode. Then, they make the scheduling plan so as to minimize the generalized travel cost associated with the selected mode. An equilibrium state is reached when no commuters can reduce their expected generalized travel costs by unilaterally changing the mode and departure time choices.

The remainder of this paper is organized as follows. In Section 2, we describe the model formulation. Section 3 examines the properties of equilibrium solutions of the model. Section 4 presents numerical results to illustrate the impact of uncertainty expectation on commuters' modal split and trip scheduling choices. Section 5 concludes the paper.

2. Model formulation

Suppose there are two alternative transport modes, a highway and a transit line, connecting a single pair of origin and destination. A population of N commuters wishes to arrive at the workplace at the work start time t^* (normalized to be zero in this study for simplicity) each morning, either by the auto mode on highway or the transit mode. For the convenience of studying, let N_A and N_T denote the number of commuters taking auto and transit modes, respectively, and $N_A + N_T = N$. Throughout the study, we assume that both modes are effectively used (i.e., $N_A > 0, N_T > 0$), which means corner solutions will not be considered. Let α represent the unit cost of travel time, β the unit cost of arrival SDE (schedule delay early), and γ the unit cost of arrival SDL (schedule delay late). According to the empirical study by Small (1992), the relation $0 < \beta < \alpha < \gamma$ always holds.

For simplicity and without loss of generality, we assume that the highway is characterized by the standard bottleneck model (Vickrey, 1969) with a fixed capacity s. The auto mode is susceptible due to the traffic incidents frequently occurring on highway, which leads to the uncertainty of travel time. To reflect the influence of this uncertainty on the travelers' mode and scheduling choices, it is further assumed that all highway users will experience an extra random time R in addition to the deterministic time. The probability density function of this random variable is known to all travelers, and takes the following form:

$$f(r) = \frac{1}{\mu} e^{-r/\mu}, \quad r \in (0, \infty), \tag{1}$$

where the parameter μ is the mean (also the standard deviation) of the distribution. The exponential distribution can set low probabilities for long delays, which is obviously reasonable in reality. For reflecting the initiative of avoiding the possible schedule delay cost induced by uncertainty, we let every auto user X, reserve a travel time budget $b(X)$ ahead of the schedule.[2]

In terms of the definition of travel time budget, the home departure time of an auto user X, without consideration of the time spent on highway, should be

$$t_{\mathrm{in}}(X) = t^* - b(X) = 0 - b(X) = -b(X). \tag{2}$$

Let the free flow travel time on highway be T_0, and the queue length that auto user X encounters be $Q(X)$. Then, his/her exit time from the highway system is

$$t_{\mathrm{out}}(X) = -b(X) + T_0 + Q(X)/s + R, \tag{3}$$

where $Q(X)/s$ is the waiting time of auto user X for passing through the bottleneck. In this way, the SDE and SDL of auto user X are as follows:

$$\begin{cases} \mathrm{SDE}(X) = \max(-t_{\mathrm{out}}(X), 0) \\ \mathrm{SDL}(X) = \max(0, t_{\mathrm{out}}(X)) \end{cases}. \tag{4}$$

For convenience of computation, like Siu and Lo (2009), we first isolate the random term R from the exit time t_{out} in Eq. (3). A pseudo exit time is defined as follows:

$$\tilde{t}_{\mathrm{out}}(X) = -b(X) + T_0 + Q(X)/s. \tag{5}$$

It is easy to see that the pseudo exit time $\tilde{t}_{\mathrm{out}}(X)$ represents the exit time of auto user X without experiencing random delay. Therefore, the expected values for SDE and SDL, in terms of the pseudo exit time $\tilde{t}_{\mathrm{out}}$, can be formulated as

$$\begin{cases} E[\mathrm{SDE}(X)] = -\int_0^{\max(-\tilde{t}_{out}(X),0)} (\tilde{t}_{out}(X) + r) f(r) dr, & \tilde{t}_{out}(X) \le -r \\ E[\mathrm{SDL}(X)] = \int_{\max(-\tilde{t}_{out}(X),0)}^{\infty} (\tilde{t}_{out}(X) + r) f(r) dr, & \tilde{t}_{out}(X) > -r \end{cases}. \tag{6}$$

There are two situations worthy to be further discussed for determining the outcomes of Eq. (6).

(i) When $\tilde{t}_{\mathrm{out}}(X) < 0$, the auto user X experiences both types of schedule delay costs at expectation level. It follows

$$\begin{cases} E[\mathrm{SDE}(X)] = -\int_0^{-\tilde{t}_{out}(X)} (\tilde{t}_{out}(X) + r) f(r) dr \\ \qquad = -\tilde{t}_{out}(X) + \mu e^{\tilde{t}_{out}(X)/\mu} - \mu \\ E[\mathrm{SDL}(X)] = \int_{-\tilde{t}_{out}(X)}^{\infty} (\tilde{t}_{out}(X) + r) f(r) dr = \mu e^{\tilde{t}_{out}(X)/\mu} \end{cases}; \tag{7}$$

(ii) When $\tilde{t}_{\mathrm{out}}(X) \ge 0$, the auto user X experiences the SDL cost only. It follows

$$\begin{cases} E[\mathrm{SDE}(X)] = 0 \\ E[\mathrm{SDL}(X)] = \int_0^{\infty} (\tilde{t}_{out}(X) + r) f(r) dr = \tilde{t}_{out}(X) + \mu \end{cases}. \tag{8}$$

Before putting forward the equilibrium travel cost functions of two modes, for convenience of understanding, the following assumptions are specially figured out:

[1] Based on the agent-based simulation results, Sunitiyoso and Matsumoto (2009) found that the traveler's expectation plays an important role in travel mode decision-making.

[2] The term 'travel time budget' defined by Lo, Luo, and Siu (2006) is the time commuters set aside for a trip, and is related to the possibility that a commuter could reach the destination at the official work start time.