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Decision Support

The economic lot-sizing problem with perishable items and consumption order preference

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ABSTRACT

We consider the economic lot-sizing problem with perishable items (ELS-PI), where each item has a deterministic expiration date. Although all items in stock are equivalent regardless of procurement or expiration date, we allow for an allocation mechanism that defines an order in which the items are allocated to the consumers. In particular, we consider the following allocation mechanisms: First Expiration, First Out (FEFO), Last Expiration, First Out (LEFO), First In, First Out (FIFO) and Last In, First Out (LIFO). We show that the ELS-PI can be solved in polynomial time under all four allocation mechanisms in case of no procurement capacities. This result still holds in case of time-invariant procurement capacities under the FIFO and LEFO allocation mechanisms, but the problem becomes $\mathcal{NP}$-hard under the FEFO and LIFO allocation mechanisms.

1. Introduction

The dynamic economic lot-sizing problem (ELS), introduced in Wagner and Whitin (1958), is described as follows. Demands for a single item over a finite and discrete planning horizon have to be satisfied by producing in a facility with no capacity restrictions. An item produced in a period can satisfy demands in that period and the following periods. Whenever there is positive production in a period, a setup has to take place, which entails a fixed setup cost. Any item produced incurs a unit production cost and any item carried to the next period incurs a unit inventory holding cost. The goal is to find a minimum cost production plan.

This basic model has been extended in several ways by, for example, considering backlogging (Zangwill, 1968), production capacities (Bitran & Yanasse, 1982 and Florian, Lenstra, & Rinnooy Kan, 1980), and inventory bounds (Atamtürk & Küçükyavuz, 2005; Hwang & van den Heuvel, 2012; Liu, 2008; Toczylowski, 1995). In all these models it is assumed that items can be kept in stock indefinitely. However, items such as agricultural products, dairy products and pharmaceutical products may deteriorate over time. For the procurement and inventory control decisions of such items, models that account for item perishability should be used.

Broadly, past research has modeled perishability in two ways. In the first one, items deteriorate continuously, and in the second one items have certain lifetimes after which they deteriorate completely. Seminal work on the first category of models includes Ghare and Schrader (1963), Covert and Philip (1973), and Philip (1974). For an extensive review of early work, see Raafat (1991) and for more recent reviews, see Goyal and Giri (2001) and Bakker, Riezebos, and Teunter (2012). Regarding the second category of models, in some it is assumed that demand follows a deterministic function (see Hwang & Hahn, 2000; Zhou & Yang, 2003), while in others demand is stochastic (see Fries, 1975; Liu & Lian, 1999; Nahmias & Pierskalla, 1973; Olsson & Tydesjö, 2010; Tekin, Gürler, & Berk, 2001). For a review of recent literature on these models, we refer the readers to Urban (2005), Bakker et al. (2012) and the related chapter in Kempf, Keskinocak, and Uzsoy (2012).

When the items are perishable, the order in which they are consumed becomes an important factor to consider. In Derman and Klein (1958) and Eilon (1962), the problem of optimally allocating items of different ages in stock to maximize total utility is considered, and the performance of a FIFO (First In, First Out) and a LIFO (Last In, First Out) allocation mechanism is evaluated. Assuming that items procured earlier deteriorate faster, FIFO (LIFO) corresponds to the mechanism where the oldest (freshest) item is issued first. In Nose, Ishii, and Nishida (1984), a perishable inventory system with stochastic procurement leadtime under both a FIFO and a LIFO allocation

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mechanism is discussed. Procurement policies when the demand rate depends on the inventory level under a FIFO and a LIFO allocation mechanism are considered in Hwang and Hahn (2000) and Zhou and Yang (2003), respectively. A model with interaction between a retailer and a supplier under both a LIFO and a FIFO allocation mechanism is studied in Hahn, Hwang, and Shinn (2004). The items have fixed lifetimes and the supplier needs to determine the amount of discount on the items so that the retailer does not return perished items. A stochastic input–output inventory system for perishable items under a FIFO and a LIFO allocation mechanism is considered in Parlar, Perry, and Stadje (2011).

ELS models that account for item perishability have been studied in the literature as well. An ELS model where a certain fraction of the inventory spoils at the end of each period is considered in Friedman and Hoch (1978). It is assumed that the fraction of items that spoil in each period increases as the items get older, so that, in a given period, items that were produced earlier deteriorate faster. A similar model with concave procurement and holding costs, where holding older items is never cheaper than holding newer items in stock, is considered in Hsu (2000). In turn, Chu, Hsu, and Shen (2005) generalize the model in Hsu (2000) by assuming economies of scale costs.

In the ELS models above, it is assumed that the allocation mechanism is FIFO and that older items deteriorate at a rate faster than newer ones (as we shall see (Section 4.6), under these assumptions there is a close link between our model and lot-sizing models with time windows). However, these assumptions may not hold in general. For example, consider the case of a retailer who orders from multiple suppliers, say, for risk mitigation. If supplier lead times are different (e.g., if they are located at different distances from the retailer), it is possible that an item may expire after a subsequently delivered item. In particular, newly received items may have an earlier expiration date than items that are already in stock. In this situation it may be suboptimal to use a FIFO policy. If the retailer is an online grocer, it might prefer to issue items in a First Expiration, First Out (FEFO) manner in order to minimize wastage. However, if the retailer is a brick-and-mortar supermarket, it loses power to control the issuance of items. In that case, when products with different expiration dates are displayed together, the customers will prefer the items with maximum remaining lifetimes. This means that the items will be issued in a Last Expiration, First Out (LEFO) manner.

Based on the above discussion, in this paper we consider a retailer who needs to solve an economic lot-sizing model with perishable items (ELS-PI), where lifetimes are general, such that an item procured later might expire earlier. We assume that items deteriorate completely after a certain expiration date, until which items remain undamaged and good for consumption. Furthermore, we consider the model both with and without capacity restrictions on the amount of procurement in each period. Because of the general lifetimes, a FIFO allocation mechanism does not imply the issuance of the oldest items first. Likewise, LIFO does not imply the issuance of the freshest items first either. Therefore, we analyze the ELS-PI explicitly under a FEFO and a LEFO allocation mechanism as well. Although the discussion above suggests that the most common settings in which items that are procured later may have an earlier expiration date are ones with multiple suppliers, for the sake of clarity in presentation, we assume that there is only a single supplier in each period to order the items from. The model with multiple suppliers was studied in the first author's doctoral thesis (Onal, 2009), and it turns out (see Section 2) that the difficulties that arise in the presence of multiple suppliers do not introduce significant changes in the analysis.

The rest of the paper is organized as follows. In Section 2, we introduce and formulate the economic lot sizing problem with perishable items (ELS-PI). In Section 3, we discuss the effect of allocation mechanisms on the optimal objective function values. We derive structural properties of optimal solutions to the ELS-PI with no procurement capacities in Section 4, and propose solution algorithms. In Section 5, we analyze the computational complexity of the ELS-PI with procurement capacities. We conclude the paper, and discuss future research subjects in Section 6.

2. The model

The ELS-PI is an economic lot-sizing problem over a discrete and finite planning horizon consisting of T periods. Demand in each period is satisfied by procurement from a single supplier with zero lead time. As soon as the items are procured, they are stored in a single location from where they are distributed to satisfy demands. The procurement amount in each period may be subject to limited capacity.

Each item in stock has a certain expiration date that depends on the period in which it is procured. Particularly, an item procured in period t expires after period $v_t \geq t$ $(t = 1, \ldots, T)$. Since we have a planning horizon of T periods, we assume that $v_t \leq T$. We assume that there is no degradation in the quality of the item as long as its expiration date has not passed. Although all items in the stock are equivalent regardless of procurement or expiration date, we allow for an allocation mechanism that defines a preference order in the allocation of items to the consumers. More formally, an allocation mechanism is characterized by a preference order of the procurement periods, where we write $i \succ j$ if and only if items procured in period i must be consumed earlier than items procured in period j. Moreover, we write $i \sim j$ to denote indifference between items procured in periods i and j. This leads to the notation $i \succsim j$ to indicate that items procured in period i are at least as preferred as items procured in period j, i.e., $i \succ j$ or $i \sim j$.

We say that a period t (partially) satisfies the demand in period i if some items procured in period t are allocated to (partially) satisfy the demand in period i. For convenience, we associate a supply interval $F(t) = \{t, \ldots, v_t\}$ with each period t, consisting of periods whose demand can be satisfied by items procured in period t. To formulate the ELS-PI, we use the following additional notation:

Parameters:

D_t = demand in period t
C_t = procurement capacity in period t
S_t = fixed setup cost of procurement in period t
p_t = unit procurement cost in period t
h_t = unit inventory holding cost in period t
c_{ti} = variable cost to satisfy one unit of demand in period i by procurement in period t
$= p_t + \sum_{j=t}^{i-1} h_j$.

Decision variables:

x_{ti} = quantity procured in period t used to (partially) satisfy demand in period i
z_{ti} = 1 if period t satisfies some demand in period i (i.e., if $x_{ti} > 0$), 0 otherwise
y_t = 1 if there is procurement in period t (i.e., if $\sum_{i=t}^{v_t} x_{ti} > 0$), 0 otherwise.

Using the parameters and decision variables above, we formulate the ELS-PI as follows:

$$\min \sum_{t=1}^{T} \left(S_t y_t + \sum_{i=t}^{v_t} c_{ti} x_{ti} \right)$$

subject to (P)

$$\sum_{t: i \in F(t)} x_{ti} = D_i \quad \text{for } i = 1, \ldots, T \tag{1}$$

$$\sum_{i=t}^{v_t} x_{ti} \leq C_t \quad \text{for } t = 1, \ldots, T \tag{2}$$