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The economic lot-sizing problem with an emission capacity constraint

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ABSTRACT

We consider a generalisation of the lot-sizing problem that includes an emission capacity constraint. Besides the usual financial costs, there are emissions associated with production, keeping inventory and setting up the production process. Because the capacity constraint on the emissions can be seen as a constraint on an alternative objective function, there is also a clear link with bi-objective optimisation. We show that lot-sizing with an emission capacity constraint is $\mathcal{NP}$-hard and propose several solution methods. Our algorithms are not only able to handle a fixed-plus-linear cost structure, but also more general concave cost and emission functions. First, we present a Lagrangian heuristic to provide a feasible solution and lower bound for the problem. For costs and emissions such that the zero inventory property is satisfied, we give a pseudo-polynomial algorithm, which can also be used to identify the complete set of Pareto optimal solutions of the bi-objective lot-sizing problem. Furthermore, we present a fully polynomial time approximation scheme (FPTAS) for such costs and emissions and extend it to deal with general costs and emissions. Special attention is paid to an efficient implementation with an improved rounding technique to reduce the a posteriori gap, and a combination of the FPTASes and a heuristic lower bound. Extensive computational tests show that the Lagrangian heuristic gives solutions that are very close to the optimum. Moreover, the FPTASes have a much better performance in terms of their actual gap than the a priori imposed performance, and, especially if the heuristic's lower bound is used, they are very fast.

1. Introduction

In recent years, there has been a growing tendency to not only focus on financial costs in a production process, but also on its impact on the environment. An important example of an environmental implication is the emission of pollutants during production. Particular interest is paid to the emission of greenhouse gases, such as carbon dioxide (CO_2), nitrous oxide (N_2O) and methane (CH_4). By now, there is a general consensus about the effect that these gases have on global warming. Consequently, many countries strive towards a reduction of these greenhouse gases, as formalised in treaties, such as the Kyoto Protocol (United Nations, 1998), as well as in legislation, of which the European Union Emissions Trading System (European Commission, 2010) is an important example.

The shift towards a more environmentally friendly production process can be caused by such legal restrictions, but also by a company's desire to pursue a 'greener' image by reducing its carbon footprint. As Vélazquez-Martínez, Fransoo, Blanco, and Mora-Vargas (2014) mention: "A substantial number of companies publicly state carbon emission reduction targets. For instance, in the 2011 Carbon Disclosure Project annual report (Carbon Disclosure Project, 2011), 926 companies publicly commit to a self-imposed carbon target, such as FedEx, UPS, Wal-Mart, AstraZeneca, PepsiCo, Coca-Cola, Danone, Volkswagen, Campbell and Ericsson."

In order to reduce their carbon footprints, companies can reduce their emissions by, for instance, using less polluting machines or vehicles, or using cleaner energy sources. One should not overlook the potential benefit that changing operational decisions has on emission reduction. As mentioned by Absi, Dauzère-Pérèz, Kedad-Sidhoum, Penz, and Rapine (2013), examples of studies at the operational level are Bektas and Laporte (2011) who consider a vehicle routing problem where the objective function consists of both travel distance and environmental costs, and Fang, Uhan, Zhao, and Sutherland (2011) who study a flow shop scheduling problem with both cycle time and environmental factors as an objective. Also in production planning there is no guarantee that minimising costs of operations will lead to low emissions. In fact, fashionable production strategies like just-in-time production, with its frequent less-than-truckload shipments and frequent

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change-overs on machines, may lead to emission levels that are far from optimal.

For these reasons, we consider a generalisation of the classic economic lot-sizing model. Besides the usual financial costs, emission levels are associated with production, keeping inventory and setting up the production process. Set-up emissions can for example originate from having fixed per-truckload emissions of an order, or from a production process that needs to 'warm up' (see e.g. Jans & Degraeve, 2004), where usable products are not created until the production process has gone through a set-up phase that is already polluting. Keeping inventory can also emit pollutants, for instance because products need to be stored in a specific way, e.g. refrigerated, or because of lighting systems in a warehouse, which emit indirectly due to their electricity consumption. The lot-sizing model that we consider in this paper minimises the (financial) costs under an emission capacity constraint. This constraint is imposed as one global restriction over all periods. This problem was introduced by Benjaafar, Li, and Daskin (2013), who integrate carbon emission constraints in lot-sizing models in several ways. They consider a capacity on the total emissions over the entire problem horizon, as we do in this paper, but also a carbon tax, a capacity combined with emissions trade, or carbon offsets (where additional emission rights may be bought, but not sold). Moreover, they study the effect of collaboration between multiple firms within a serial supply chain. They examine how the values of the problem parameters, as well as the parameters of regulatory emission control policies, affect costs and emissions. They assume that all cost and emission functions follow a fixed-plus-linear structure, and no attention is paid to finding good solution methods.

In our paper, we study a lot-sizing problem with an emission constraint under concave cost and emission functions. We show that this problem is $\mathcal{NP}$-hard, even if only production emits pollutants (linearly). We will see that this model is also capable of handling multiple production modes. Furthermore, we develop several solution methods. First, we develop a Lagrangian heuristic that finds both very good solutions and a lower bound in $\mathcal{O}(T^4)$ time, where T is the number of time periods. We also prove several structural properties of an optimal solution that we use while working towards a fully polynomial time approximation scheme (FPTAS). As a first step, a pseudo-polynomial algorithm is developed in case the costs and emissions are such that the single-sourcing (or zero inventory) property is satisfied in an optimal solution, that is, there is no period with both strictly positive production and strictly positive incoming inventory. This pseudo-polynomial algorithm is then turned into an FPTAS, which, in turn, is generalised to deal with costs and emissions that do not satisfy the single-sourcing property. We expect that this technique to construct a pseudo-polynomial algorithm and an FPTAS can be applied to more problems where one overall capacity constraint is added to a problem for which a polynomial time dynamic program exists.

Special attention is paid to an efficient practical implementation of these algorithms. This includes a combination of the lower bound that is provided by the Lagrangian heuristic with an FPTAS. This combination results in excellent solutions within short computation times, as becomes clear from the extensive computational tests of all algorithms that have been carried out for this paper. Besides that, our algorithms do not only have an a priori gap (ε), but they also produce a (smaller) a posteriori gap. To reduce this gap even further, we develop an improved rounding technique, which we think can be applied to other FPTASes of the same type. Furthermore, if we compare the algorithms' solutions to the optima, we see that the gaps are even much smaller.

The model is more general than it looks at first sight, since the 'emissions' in the 'emission constraint' that we consider do not necessarily need to literally refer to emissions. They can be *any* kind of costs or output, other than those in the objective function, related to the three types of decisions (i.e., set-up, production and inventory). This makes the relationship with bi-objective lot-sizing clear. In multi-objective optimisation (and bi-objective optimisation in particular), one is usually interested in the set of Pareto optimal solutions or Pareto optimal set for short. Theoretically, finding the optimal costs for all possible emission capacities would result in finding the Pareto optimal set. The multi-objective lot-sizing problem is studied in more detail by Van den Heuvel, Romeijn, Romero Morales, and Wagelmans (2011), who divide the horizon in several blocks, each with its own objective function. The case with one block of length T corresponds to our problem (with fixed-plus-linear costs and emissions). In our paper, we will show that we can find the Pareto optimal set in pseudo-polynomial time, if the costs and emissions are such that the single-sourcing (zero-inventory) property is satisfied.

Besides the works of Benjaafar et al. (2013) and Van den Heuvel et al. (2011), there are other papers that integrate carbon emission constraints in lot-sizing problems. Absi et al. (2013) introduce several lot-sizing models with multiple production modes and a constraint on the average amount of emission per unit produced, which may be on a periodic, cumulative, global (as we have) or rolling horizon basis. Although the models are similar to our work, the main difference is the type of emission constraint: we have a bound on the total emission, which acts as a kind of capacity constraint, while Absi et al. (2013) have a bound on the average emission per unit produced. Furthermore, in Absi et al. (2013) the emission is linear in the amount produced, while our emission functions are more general with emissions caused by setups and holding inventory. Since our model can also handle multiple production modes, the models coincide (except for the emission function structure) in case of a global emission constraint. Vélazquez-Martínez et al. (2014) study the effect of different levels of aggregation to estimate the transportation carbon emissions in the economic lot-sizing model with backlogging. Heck and Schmidt (2010) discuss lot-sizing with an 'eco-term', which they solve heuristically with an 'eco-enhanced' Wagner–Whitin and Part Period Balancing algorithm, with the possibility of 'eco-balancing'. Other papers approach the emission problem from an EOQ (economic order quantity) point of view, such as Chen, Benjaafar, and Elomri (2013), Hua, Cheng, and Wang (2011), and Bouchery, Ghaffari, Jemai, and Dallery (2012).

The remainder of this paper is organised as follows. The next section provides a formal, mathematical definition of the lot-sizing problem with a global emission capacity constraint. In Section 3, we show that this problem, as well as a variant with two production modes, is $\mathcal{NP}$-hard under quite general conditions. Furthermore, we prove several structural properties of an optimal solution, which are used by the algorithms that are introduced in Section 4. Section 4.1 gives a Lagrangian heuristic. Sections 4.2 and 4.3 present a pseudo-polynomial algorithm, respectively FPTAS, for what we will define as *co-behaving* costs and emissions, that is, cost and emission functions which move in the same direction. An FPTAS for general costs and emissions is derived in Section 4.4. The combination of the heuristic and FPTASes is discussed in Section 4.5. Section 5 describes the results of the extensive computational tests and the paper is concluded in Section 6. In an online supplement, the proofs of several theorems from this article are given, as well as pseudocode for the Lagrangian heuristic, additional information on backtracking in one of the FPTASes, and tables with detailed results.