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Decision Support

Planning safe navigation routes through mined waters

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ABSTRACT

In this paper we investigate the problem of finding a safe transit of a ship through areas threatened by sea mines. The aim is to provide decision-making support by a tool that can be integrated into a naval command and control system. We present a route finding algorithm which avoids regions of risk higher than a given threshold. The algorithm takes into account the technical and operational restrictions of the ship's movement. It allows to minimize the route length, the traveling time, the number of maneuvers, or other objectives.

The basic idea is to embed a network in the operational area and compute a least-cost path. Instead of using a regular grid graph which strongly restricts the types of maneuvers and necessitates a path smoothing after optimization, we design a network which is especially tailored to the maneuverability of the vessel. Each path in this network represents a continuous-curvature track based on combinations of clothoids and straight line segments. The approach allows a large variety of maneuvers, hence high-quality solutions are achievable provided a sufficiently dense network.

1. Introduction

The rapid increase in the proliferation of sea mines presents a unique challenge to maritime security. Sea mines are self-contained explosive devices placed in water to destroy surface ships or submarines. Since mines are inexpensive they form an effective means to deny the use of large areas. For over a century, they have been used as a strategic weapon in military conflicts to blockade ports, waterways or other maritime zones. The naval forces must be prepared to counter the danger posed by mines. This, in particular, includes to find safe passages in order to sustain commercial and military shipping operations.

Planning a navigation route through an area threatened by sea mines is closely related to the classical problem of finding a collision-free path of a vehicle in an environment with obstacles. The latter, also known as path planning or trajectory planning problem, has long been the subject of intensive research. We refer to the reviews of Latombe (1991) and LaValle (2006).

Numerous ideas have been investigated to resolve the path planning problem. The most promising include the potential field method, cell decomposition, the roadmap method, the mass-spring-damper method, mixed integer linear programs, and several network-based methods with discretizations of the space by visibility graphs, Voronoi diagrams or regular grids. Newer approaches include the rapidly-exploring random tree method, model predictive control methods, and mathematical programming methods. Comprehensive reviews are given by Goerzen, Kong, and Mettler (2010) and Ferguson, Likhachev, and Stentz (2005).

While previous research mainly focused on vehicles moving on ground (mobile robots, autonomous vehicles) or in air (unmanned aerial vehicles, drones, autonomous helicopters), less work has been published for sea vehicles. Although the problems are closely related the proposed algorithms are not directly applicable since, from a technological point of view, ships behave quite differently than ground vehicles or aircraft. Moreover, shipping may imply specific operational requirements.

An important fact is that continuous methods can only find locally optimal solutions which may be far from globally optimal. On the other hand, network-based methods give an approximate problem, but means that one can find a globally optimal solution (see e.g. Zabarankin, Uryasev, & Pardalos, 2002). Furthermore, network-based methods appear more promising for realizing specific technological and operational constraints.

Fagerholt, Heimdal, and Loktu (2000) present an algorithm to find a shortest route for a ship located at sea to a destination port, in the presence of polygonal obstacles. It relies on the partial computation of a visibility graph and solves the problem using two variants of the algorithm of Dijkstra.

In a series of papers, Piatko, Diehl, McNamee, Resch, and Wang (2002), Piatko, Priebe, Cowen, Wang, and McNamee (2001) and Resch, Piatko, Pineda, Pistole, and Wang (2003) present path

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planning techniques to find low-risk paths of a ship through a minefield. They discretize the operational area by evenly distributing vertices, with eight edges emerging from each vertex (along the axes and the diagonals), hence generating a regular grid graph. The authors develop probabilistic risk models and identify different path quality attributes. The problem is then solved by defining appropriate costs on the edges of the graph and calculating a least-cost path.

A similar method has been suggested by Bekker and Schmid (2006). They also use a regular grid graph, and remove all edges which lie within some safety radius around the mines. The remaining edges get as a cost either the length, or a risk value depending on the distance to the mines, or a mixture of both. Again a least-cost path has to be determined. Their approach has been extended by Li (2009) who added longer edges to the graph in order to improve the solution.

A drawback of the above methods is that important factors including the characteristics of the ship (maneuverability, turning circle, etc.) or restrictions which are implied by operational reasons (e.g. no sharp maneuvers due to passenger or cargo safety) are not considered. Each method provides a polygonal line, possibly with sharp turns or extreme maneuvers, which the ship may not be able to follow.

Turn constraints have been successfully implemented in a recent research of Ari, Aksakalli, Aydoğdu, and Kum (2013) where the goal is to find a shortest path through an area with obstacles. Their approach is based on a regular grid as above, however, they use a vertex replication technique where each vertex is split into copies labeled by the direction the ship is coming from. This incorporates navigation history into the present location and allows to impose realistic ship turn constraints. Moreover, the operational area can be discretized at any desired resolution. Indeed, there is still room for improvement since ship maneuvers are limited to 45° turns with full rudder deflection (which might imply that the solution is far from optimal) and a path smoothing is required after optimization (which might cause collision with an obstacle if not provided with a buffer zone).

The aim of our work is to develop an algorithm which finds a safe route through a mined area. Particular attention will be given to the technical capabilities and operational restrictions of the ship's movement. Moreover, the maritime environment including shallows, islands, obstacles, etc., will be considered. The algorithm is intended to be integrated as a decision-making tool into a naval command and control system.

We propose a network optimization approach, focusing on the generation of a special randomized network which is adapted to the maneuverability of the ship. The paths in the network correspond to smooth (continuous-curvature) tracks consisting of clothoids and straight line segments. The approach allows to set restrictions on the track including (non-symmetric) turn radius constraints, thus avoiding too sharp maneuvers, but also other restrictions such as the exclusion of long-lasting weak maneuvers (which might be undesirable due to operational reasons). The change of velocity during turning maneuvers is taken into account. The random structure of the network allows a large variety of maneuvers of different strengths and without turn degree limitations.

By applying standard shortest-path methods, we can determine routes which avoid regions of risk higher than a given threshold. With appropriate costs defined on the edges, different objectives can be considered such as minimizing route length, traveling time, the number of maneuvers, or some compromise of these. Like all network-based methods, our approach is of a heuristic nature. On the other hand, the quality of the solutions can be continually improved by increasing the size of the network, as illustrated in Section 5.

In the next section, a mathematical model of the ship's movement is presented, along with a brief description of the threat model and the optimization problem. Section 3 contains the basic idea of the network generation and the route planning algorithm. In Section 4 we discuss how to speed up the algorithm by using efficient intersection procedures, and by separating the planning into preprocessing and online planning. Computational results are presented in Section 5. We conclude with final remarks in Section 6.

2. Mathematical modeling

2.1. Ship track model

Our first task is to establish a realistic model of the movement of a ship. From an operational point of view, a planned ship track should fulfill certain requirements. First, the track should be easy to navigate, i.e. it should allow easy handling and control of the ship by the crew. To the extent possible, long-lasting maneuvers and extreme course alterations should be avoided. Finally, the track's curvature should be continuous in order to guarantee a smooth navigation. This is because following a planned track should not unduly burden the ship and the crew. These aspects become crucial in transporting people or dangerous goods. Comfort and safety can only be ensured for continuous-curvature trajectories.

A *clothoid*, also known as *Euler* or *Cornu spiral*, is a curve whose curvature changes linearly with its length (Abramowitz & Stegun, 1972). It is well known that a vehicle following the curve at constant speed will have a constant change of centripetal acceleration. Hence, the clothoid is a perfect representation of a vehicle traveling at constant speed with a constant rate of change of the steering angle (Kanayama & Miyake, 1985). The linear steering motion makes such a track comfortable for vehicle and crew.

It has been shown in Mohović, Mohović, and Rudan (2012) that a clothoid very well approximates the curve of a turning ship up to a turning angle of about 100–120°, although the speed is not constant. In the first phase of the maneuver, when the rudder is deflected, the track is close to a straight line since the ship starts turning very slowly. Then the curvature increases and the track gradually turns into a circle. During this part of the turn, the speed of the ship decreases. If the angle exceeds 120° the ship moves with constant speed in a circle with constant radius. The characteristic phases of a turning ship and the approximating clothoid are sketched in Fig. 1. Our goal is to realize the ship track as a combination of straight line segments and clothoids. The transitions between the track segments have to be smooth.

A clothoid can be expressed as a parametrized curve

$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad 0 \leqslant t \leqslant T$$

with domain $[0, T]$ and

$$x(t) = a\sqrt{\pi}\int_0^t \cos\frac{\pi\tau^2}{2}\,\mathrm{d}\tau, \quad y(t) = a\sqrt{\pi}\int_0^t \sin\frac{\pi\tau^2}{2}\,\mathrm{d}\tau.$$

It is well known and straightforward to verify that the arc length and the curvature of the curve are

$$s = a\sqrt{\pi}t, \quad \kappa = \frac{\sqrt{\pi}}{a}t,$$

respectively. Hence

$$\kappa = \frac{1}{a^2}s,$$

i.e. the curvature is proportional to the arc length. The parameter a represents a scale factor which defines the rate of change of the curvature and hence the size of the clothoid. The Fresnel integrals