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Decision Support

Global approximation to arbitrary cost functions: A Bayesian approach with application to US banking ☆

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ABSTRACT

This paper proposes and estimates a globally flexible functional form for the cost function, which we call Neural Cost Function (NCF). The proposed specification imposes *a priori* and satisfies globally all the properties that economic theory dictates. The functional form can be estimated easily using Markov Chain Monte Carlo (MCMC) techniques or standard iterative SURE. We use a large panel of U.S. banks to illustrate our approach. The results are consistent with previous knowledge about the sector and in accordance with mathematical production theory.

1. Introduction

Cost analysis has been an indispensable tool for economists, managers and engineers in evaluating firm's performance. In this context, reliable estimates of cost functions are of great importance because they can assist in accurate decision making. Also, other related types of analysis such as efficiency evaluation, require the estimation of a cost function as a first step towards the final evaluation of firms.

According to Henderson and Parmeter (2009), non-parametric methods are desirable from an applied standpoint since economic theory rarely provides closed form solutions to the models of interest. However, the ability to restrict a model to mimic economic theory underlying its creation is also a key feature in choosing an estimation method, so that appropriate tests of the model's assumptions may be generated. In this context, according to Gallant and Golub (1984), one has to work with functions that are parametrically rich enough in order to be capable of reproducing all the properties that economic theory dictates. If this is not the case, then any restrictions imposed on parameters could limit the generality of inference. Thus, here we exploit the semi-parametric nature of Artificial Neural Networks (ANNs), which are rich in parameters, in order to impose all the properties that mathematical economic theory dictates regarding the cost function. In fact, our proposed method is consistent with other researchers, such as the work of Du, Parmeter, and Racine (2013) in the sense that it is able to handle multiple general shape constraints for multivariate functions, or the work of Kuosmanen and Kortelainen (2012) and Kuosmanen and Johnson (2010) who employ non-parametric least squares methods in order to ensure convexity.

In this work, we propose and estimate a new cost function based on AANs which we call the Neural Cost Function (NCF) that has considerable advantages: (a) It is a global approximation to *any* arbitrary cost function; (b) it satisfies all the properties dictated by economic theory *globally* and not only over the set of inputs and outputs where inferences are drawn; (c) it provides a very good fit to real-world data because it is data driven; (d) it allows for *arbitrary* returns to scale; (e) it is relatively simple to estimate. Despite the fact that a small fraction of these desirable properties might also be possessed by one or other of the already known specifications, neither possesses all of them simultaneously.

An empirical application investigating the model's performance using Bayesian techniques illustrates our technique based on a large data set on all US commercial banks. We believe that the

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results of this study suggest that the proposed cost function is a useful approach for expanding conventional cost analyses.

The paper is organized as follows: Section 2 offers a brief discussion of the theoretical framework; Section 3 introduces the proposed cost function; in Section 4 the measures of interests are presented; Section 5 sets out the empirical analysis; Section 6 concludes.

2. Theoretical framework

Despite the fact that the so-called production function is a totally different thing from a cost function, there is a close relationship between them in the sense that "the cost function contains essentially the same information that the production function contains", the so-called *duality* of production and cost (Varian, 1992), which was established by Shephard (1953) and Diewert (1971). In simple words, *duality* implies that, for given values of the parameters of the cost function, we obtain the parameters of the production function and *vice versa* (e.g. Stewart, 2005) and has found countless applications in the literature (e.g. Yair, 1996).

In this framework, it is of paramount importance to be able to find a suitable functional form for the cost function, a process which is not an easy task. Thus far, several functional forms for the cost function have been proposed in the relevant literature such as the Cobb-Douglas specification (Cobb & Douglas, 1928), and the Translog functional form (Christensen, Jorgenson, & Lau, 1971). However, both of these forms suffer from serious drawbacks, the two most important of which are the following: First, the Cobb-Douglas specification uses a naïve mathematical function, despite the fact the real-world data are quite complicated. Second, both specifications do not comply with the desired properties dictated by mathematical economic theory, such as monotonicity, curvature, and homogeneity conditions.

In this context, some researchers have developed numerical (Gallant & Golub, 1984) and Bayesian techniques (O' Donnell & Coelli, 2005) for imposing the aforementioned conditions that have provided very satisfactory results. However, when working in a rigorous context in accordance with modern economic theory, it is of great importance to estimate functional forms that satisfy the aforementioned conditions *globally,* a research topic which is "one of the most vexing problems applied economists have encountered" (Diewert & Wales, 1987) and remains "one of the most difficult challenges faced by empirical economists" (Terrell, 1996).

It should be noted that, so far, no empirical study has imposed all the theoretical properties dictated by production theory, despite the prominent efforts of some researchers, such as O' Donnell and Coelli (2005). Actually, only a small number of studies even report the degree to which the estimated functions satisfy these conditions (Reinhard & Thijssen, 1998). In fact, O' Donnell and Coelli (2005) in a prominent paper induced, in a Translog output distance function, three (3) of the properties that production theory dictates, namely: (a) homogeneity, (b) monotonicity, and (c) convexity by imposing the needed restrictions on the set of the translog parameters. Our paper is fundamentally different from the work of O' Donnell and Coelli (2005) in that we propose a Neural Network cost function which is formally proved to be a global approximator to any arbitrary cost function satisfying by construction, i.e. globally and *a priori*, all the theoretical properties dictated by economic theory and not only some of them over the set of inputs and outputs where inference is drawn, as e.g. in O' Donnell and Coelli's (2005) seminal paper.

In this work, we propose and estimate the Neural Cost Function (NCF) which is based on ANNs. The non-linear and non-parametric nature of ANNs makes them very attractive for the estimation of a cost function given that the theoretical relationship is not known *a priori* (Zhang & Berardi, 2001). Of course, a serious drawback of ANN's is that they are a-theoretical, in the sense that their estimated coefficients are not easily interpreted. However, their advantages make them clearly extremely useful in contexts requiring increased flexibility and limited intuition. The intrinsically nonlinear character of ANNs is helpful because it permits us to obtain a sufficiently flexible functional form which is capable of approximating any arbitrary cost function while enabling the *a priori* imposition of the properties dictated by the theory. Contrary to widely used local approximations such as the Translog (Christensen et al., 1971), the generalized Leontief (Diewert, 1971) or the symmetric McFadden form (Diewert & Wales, 1987), the NCF is a global approximation to the unknown function. The Fourier flexible form (Gallant, 1982) is also a global approximation but it requires a large number of parameters.

Formally speaking, ANNs relate an output variable Υ to certain input variables $X' = [X_1, \ldots, X_n]$. The input variables are combined to form K intermediate variables or projections $Z_1, \ldots, Z_K$ where:

$$Z_\kappa = X'\beta_\kappa, \quad \kappa = 1, \ldots, K$$

And $\beta_\kappa \in \mathbb{R}^K$ are parameter vectors. The intermediate variables are combined nonlinearly to produce Υ:

$$Y = \sum_{\kappa=1}^{K} a_\kappa \varphi(Z_\kappa)$$

where φ is an activation function, the a_κ's are parameters and K is the number of nodes (Kuan & White, 1994). By combining simple units with intermediate nodes, ANNs structures are capable of approximating any smooth function (Chan & Genovese, 2001). As demonstrated in Hornik, Stinchcombe, and White (1989, 1990), ANNs provide good approximations to a large class of arbitrary functions while keeping the number of parameters to a minimum. In fact, they are universal approximations of functions including their own derivatives (Hornik et al., 1990).

3. Neural Cost Function (NCF): Mathematical properties

In this section, we propose a novel cost function specification, the Neural Cost Function (NFC), and formally prove that it constitutes a global approximation to any arbitrary cost function. We start by offering some useful definitions and then we proceed by defining the proposed NCF formally.

Definition 1 (*Cost Function*). The cost function is defined as: $C(p, y) = min_x\{px : x \in DomV \text{ and } p \in \mathbb{R}^N_+\}$, where $y \in \mathbb{R}^M_+$ denote all the output vectors, $p \in \mathbb{R}^N_+$ denote all the positive input price vectors and $DomV = \{y \in \mathbb{R}^M_+ : V(y) \neq \varnothing\}$ denotes the effective domain of the production set $V(y) = \{y \in \mathbb{R}^M_+\}$.

Remark 1. The cost function is defined for all possible output vectors, $y \in \mathbb{R}^M_+$, and all positive input price vectors $p \in \mathbb{R}^N_+$.

Remark 2. The cost function does not exist if there is no possible way to produce the output in question. In what follows, we will make the assumption that the production set $V(y) = \{y \in \mathbb{R}^M_+\}$, is closed and bounded.

Next, based on the definitions of open set, open covering, compact set, dense set and closure (A.1–A.5, Mathematical Appendix) we proceed by stating an important result (Theorem 1).