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Innovative Applications of O.R.

Optimization-based heuristics for underground mine scheduling

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ABSTRACT

Underground mine production scheduling possesses mathematical structure similar to and yields many of the same challenges as general scheduling problems. That is, binary variables represent the time at which various activities are scheduled. Typical objectives seek to minimize costs or some measure of production time, or to maximize net present value; two principal types of constraints exist: (i) resource constraints and (ii) precedence constraints. In our setting, we maximize "discounted metal production" for the remaining life of an underground lead and zinc mine that uses three different underground methods to extract the ore. Resource constraints limit the grade, tonnage, and backfill paste (used for structural stability) in each time period, while precedence constraints enforce the sequence in which extraction (and backfill) is performed in accordance with the underground mining methods used. We tailor exact and heuristic approaches to reduce model size, and develop an optimization-based decomposition heuristic; both of these methods transform a computationally intractable problem to one for which we obtain solutions in seconds, or, at most, hours for problem instances based on data sets from the Lisheen mine near Thurles, Ireland.

1. Introduction and literature review

Metallic, or hard rock, mining is separated into two principal categories: (i) surface mining and (ii) underground mining (Fig. 1). We compare the two mining approaches in Table 1. When the ore is located close to the earth's surface, the most productive and economically efficient mining method is *open pit*, a form of surface mining. With this method, notional three-dimensional blocks of material are extracted from an ore body, regardless of whether they meet the *cut-off grade*, i.e., the percentage mineral content at which the material is deemed profitable, declared ore, and processed. Material below the cut-off grade is termed waste and discarded. Consequently, a significant factor in the viability of the open pit approach is the ratio of extracted ore to extracted waste.

Despite its relatively lower infrastructure cost, surface mining becomes cost-prohibitive when the ratio of extracted waste to ore becomes too high. Below ground, trucks haul ore throughout miles of tunnels. Crushed ore is conveyed to the surface for refinement at the mill. Tailings (i.e., waste produced by the mill) are disposed of in a tailings pond. With targeted extraction methods that minimize costly waste production, underground miners seek to extract only blocks of material that will be processed into mineral concentrate. Underground mining is also suitable for mineral deposits located in environmentally sensitive areas, where high reclamation costs would be associated with an open pit operation. However, while they minimize waste production and the environmental footprint, these underground operations are also more complex, have higher extraction costs, and are far more dangerous than surface mines.

The mine production scheduling problem requires planners to select and schedule blocks for extraction in a sequence that maximizes or minimizes a specific goal, e.g., minimizes deviations from planned production targets. We use binary variables to represent the time at which a block is scheduled. Two principal types of constraints exist: (i) resource constraints, which limit the number of activities committed to a time period based on the availability of a given resource and on the amount of that resource required to perform the activity and (ii) precedence constraints, which dictate the order in which activities must be completed.

The quality of the production schedule used in the exploitation stage of mining depends, in part, both on the accuracy of the model (and solution procedure), and on the quantity of alternative scheduling scenarios that planners examine; while mine planning software enables the examination of a greater number of schedules, the complex and mine-specific nature of the operations and the

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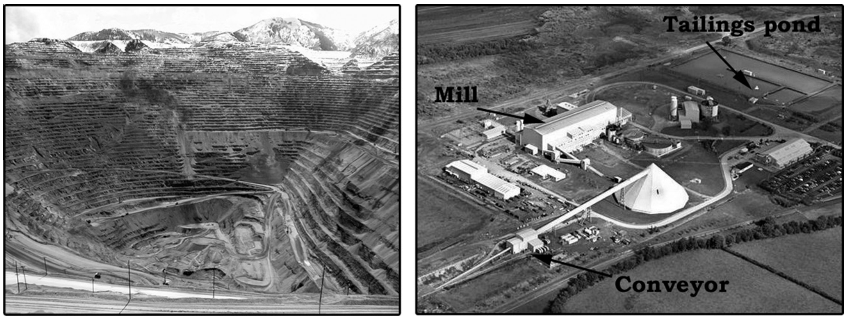

Fig. 1. Surface mining (left) can be used when ore is located close to the earth's surface. The small footprint of the facilities at the Lisheen underground mine in Ireland (right) provides little evidence with which to judge the size of the mine.

Table 1
A broad comparison of open pit and underground mining approaches; please note that these are generalizations.

Attribute	Open pit	Underground
Complexity	Less complex	More complex
Waste mining	Very high	Low
Stockpiling ore	Yes	No
Environmental disruption	Large footprint	Small footprint
Safety	Relatively safe	Dangerous
Extraction costs	Low	High
Reclamation costs	High, if reclamation required	Relatively low

combinatorial nature of the production scheduling problem in many cases preclude both the generation of near-optimal schedules and the quick examination of the entire set of alternatives, especially for schedules with a large number or activities and/or a long time horizon with fine fidelity. Consequently, mine production plans can be far from optimal. While better models are being developed, solution techniques are being refined, and hardware and software continue to improve (Newman, Rubio, Caro, Weintraub, & Eurek, 2010), production scheduling problems, particularly for underground mining operations, continue to challenge researchers.

Applications of operations research to mining have concentrated on strategic and tactical planning decisions at the development and exploration stages in the life cycle of a mine (Newman et al., 2010; Topuz & Duan, 1989). Strategic planning is concerned with long-term decisions that affect the value of the mining venture, such as: (i) determining the most economical pit size for a surface mine, or the *ultimate pit*; (ii) locating the processing plant (i.e., the mill) and other facilities; (iii) selecting machines and equipment; (iv) designing infrastructure, e.g., the number and placement of ramps, roads, and shafts; and (v) scheduling production in the long term. Over shorter horizons, tactical planning considers the decisions required to realize the objectives of the strategic plan. Examples include: (i) routing trucks; (ii) blending ore at the mill; and (iii) scheduling production in the short term.

Over the past fifty years, operations research associated with mine planning has focused primarily on open pit mining. The majority of this work is a derivative of Lerchs and Grossmann's (1965) network-based solution approach for the *ultimate pit limit* problem, e.g., Underwood and Tolwinski (1998) and Ramazan (2007). The Lerchs–Grossmann algorithm forms the basis for many of the commercial software packages used in industry to schedule open pit mine production, e.g., Whittle (GEOVIA, 2012), and Vulcan Optimizer (Maptek, 2012). A recent study includes (Kawahata, Schumacher, & Hufford, 2013), who compare in-house and commercial optimizers for Newmont's Nevada operations. Unlike open pit mining, there is not a “core” optimization model for underground mine planning, and there is virtually no commercial optimization software available.

The application of optimization to underground mine production planning begins with Williams, Smith, and Wells (1973), who demonstrate that a linear programming approach can be used to solve high-level strategic problems. Their method employs continuous variables and, consequently, cannot enforce the logic to schedule production at an operational level. Seventeen years later, Chanda (1990) employs integer programming when creating a schedule for six consecutive work shifts at a copper mine in Zambia. He circumvents the difficulty in solving multi-period integer programs by combining mixed integer programming (MIP) and simulation to produce a solution.

Maximizing the net present value of a copper mine for a 17-period time horizon, Trout's (1995) model incorporates sequencing, production, and backfilling constraints. He produces a schedule by introducing variable restrictions that yield a tractable problem. Winkler (1996) outlines the suitability of mixed integer programming to underground mine scheduling problems. However, she also illustrates the exponential complexity associated with this approach when producing a schedule for a multi-period time horizon. She encompasses single-period MIP solutions within a simulation routine to produce a multi-period schedule for a German coal mine. Maximizing discounted ore revenue, Carlyle and Eaves (2001) apply mixed integer programming to schedule production at an underground platinum and palladium mine for a number of mine expansion scenarios. The authors solve their model, that incorporates development and extraction decisions, to generate schedules for ten time periods, but do not provide details of any special solution methods.

Ataee-pour (2005) reviews existing algorithms, which he categorizes as exact or heuristic, for optimizing stope layout; the layout is analogous to the ultimate pit limit in open pit mining, and defines the design in underground mining. In developing the various design algorithms, the author demonstrates how the constraints involved in different underground mining methods are considered. Nehring, Topal, and Little (2010) solve a mixed integer programming model for instances of varying sizes (numbers of stopes) in a sublevel stoping mine. By recognizing that in this mine, the activities of development, drilling, extraction and backfilling are done continuously and in sequence for a given area, the authors combine four variables into one; the result is a substantial decrease in solution time without compromising solution quality. Little, Knights, and Topal (2013) integrate stope design and production scheduling decisions. Not surprisingly, the authors show that when these two types of decisions are integrated, a better objective function value is obtained. Of course, the stope design options must be enumerated, which leads to larger and more difficult integrated models. While we take stope design as given, our production scheduling model is more complicated than those mentioned in