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Discrete Optimization

Two-phase branch-and-cut for the mixed capacitated general routing problem

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ABSTRACT

The Mixed Capacitated General Routing Problem (MCGRP) is defined over a mixed graph, for which some vertices must be visited and some links must be traversed at least once. The problem consists of determining a set of least-cost vehicle routes that satisfy this requirement and respect the vehicle capacity. Few papers have been devoted to the MCGRP, in spite of interesting real-world applications, prevalent in school bus routing, mail delivery, and waste collection. This paper presents a new mathematical model for the MCGRP based on two-index variables. The approach proposed for the solution is a two-phase branch-and-cut algorithm, which uses an aggregate formulation to develop an effective lower bounding procedure. This procedure also provides strong valid inequalities for the two-index model. Extensive computational experiments over benchmark instances are presented.

1. Introduction

This paper presents a new exact algorithm for the *Mixed Capacitated General Routing Problem* (MCGRP) based on branch-and-cut (B&C). The MCGRP generalizes the single-vehicle and multiple-vehicle *General Routing Problems* (GRPs) and the *Capacitated Arc Routing Problem* (CARP).

GRPs constitute a class of vehicle-routing problems, in which a single vehicle or a fleet of vehicles must serve both a subset of links and a subset of vertices of a given graph. GRPs have interesting practical applications, prevalent in waste collection, postal delivery and school bus routing. For instance, in an urban waste collection plan, the collection along a street may be modeled by means of links that must be traversed, whereas the collection occurring in specific points (e.g., hospitals or multi-storey apartment blocks) may be modeled by means of vertices that must be visited. Similarly, in the postal delivery services, depending on their demand and dispersion, customers may be modeled as individual vertices or groups of customers as street segments (edges or arcs). Finally, in school bus routing, several children living on the same street may be picked up either by stopping close to each ones home, implying a service on the respective street segments, or groups of them may walk from their home to a specific bus stop imposing just one stop.

The GRP was introduced by Orloff (1974) and shown to be $\mathcal{NP}$-hard by Lenstra and Rinnooy Kan (1976). Most works refer to the uncapacitated case. Specifically, Letchford (1996, 1999) and Corberán and Sanchis (1998) proposed valid inequalities for the GRP polyhedron. For the same problem, Corberán, Letchford, and Sanchis (2001) described a cutting-plane algorithm based on several classes of facet-inducing inequalities. Reinelt and Theis (2008) studied the 0/1-polytope associated with the uncapacitated GRP defined over a connected and undirected graph. The contribution of Corberán, Romero, and Sanchis (2003) for the GRP defined on a mixed graph was a new integer programming formulation and a partial description of the related polyhedron. They reported remarkable computational results obtained by a cutting-plane algorithm. Corberán, Mejía, and Sanchis (2005) considerably improved this algorithm by defining a new family of facet-defining inequalities and new separation procedures. Blais and Laporte (2003) proposed a transformation in order to solve the uncapacitated GRP defined over directed, undirected and mixed graphs. The GRP is transformed into an equivalent traveling salesman problem or rural postman problem and solved by means of available exact algorithms. The approach does not work equally well in all cases; it works best on directed problems and on mixed problems, in which the number of edges is relatively small. The

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uncapacitated GRP was also modeled by resorting to windy graphs. Corberán, Plana, and Sanchis (2007, 2008) presented a strong windy general routing polyhedron description and designed a powerful B&C algorithm able to solve a large number of benchmark instances.

The basic multiple-vehicle routing problem is the *Capacitated Vehicle Routing Problem* (CVRP, see Toth & Vigo, 2002, 2014), in which the demand occurs only at vertices. On the contrary, arc routing problems (ARPs, see Corberán & Laporte, 2014; Dror, 2000) are GRPs in which no vertices have to be serviced. While CVRPs are defined on complete graphs, ARPs share with GRPs that they are defined on incomplete (often sparse) graphs, which are either undirected, directed, mixed, or windy.

Important contributions for the mixed CARP have been given by Belenguer, Benavent, Lacomme, and Prins (2006). They presented a linear formulation, developed a lower bounding procedure based on valid inequalities, and described some upper bounds obtained through three constructive heuristics and a memetic algorithm. Gouveia, Mourão, and Pinto (2010) described a compact flow-based model for the mixed CARP and derived an aggregate lower bounding model. Moreover, they introduced a set of valid inequalities for the linear programming relaxation of the integer model and presented promising computational results.

Note that GRPs can be transformed into CARPs by adding loops, i.e., edges $\{i, i\}$ or arcs (i, i) to the underlying graph whenever in the GRP instance a vertex i has to be serviced. The edge or arc receives the same demand as the vertex that it substitutes. In this sense, then the mixed CARP and the MCGRP can be considered identical, at least if the mathematical formulation and solution approach is capable of handling loops. To the best of our knowledge, this equivalence has not yet been utilized.

The problem studied and solved in the paper at hand is the MCGRP. It may cause confusion that sometimes the MCGRP is referred to as *Capacitated General Routing Problem on mixed graphs* (CGRP or CGRP-m) and *Node, Edge and Arc Routing Problem* (NEARP). There exist lower bounding procedures and tailored exact algorithms for its solution (Bach, 2014; Bach, Hasle, & Wøhlk, 2013; Bosco, Laganà, Musmanno, & Vocaturo, 2013; Gaze, 2013; Gaze, Hasle, & Mannino, 2013). Other studies present non-exact approaches tackling the problem. Particularly, Pandit and Muralidharan (1995) described a heuristic procedure which starts with a sub-graph obtained from the original one by considering only the links that must be traversed and the vertices that must be visited. Since the sub-graph is generally disconnected, the connection is reached by adding to it the shortest paths linking two vertices of disjoint connected components. The sub-graph is then converted into a Eulerian graph which admits a giant tour. A feasible solution is obtained by cutting the giant tour into smaller tours satisfying the capacity constraints. Gutiérrez, Soler, and Hervás (2002) introduced an alternative procedure, based on the partition-first-route-next paradigm, improving previous results. Prins and Bouchenoua (2005) described a memetic algorithm for the MCGRP. Bosco, Laganà, Musmanno, and Vocaturo (in press) introduced a matheuristic algorithm for the MCGRP where the exact algorithm of Bosco et al. (2013) is incorporated in some steps of a neighborhood search. Hasle, Kloster, Smedsrud, and Gaze (2012) carried out a computational study on three large scale MCGRP datasets. Finally, an extension of the MCGRP was tackled by Bräysy, Martínez, Nagata, and Soler (2011).

We propose an alternative exact approach to solve the MCGRP which combines beneficial ingredients from existing procedures in an effective way. The novelty of the approach substantially comprises two aspects. First, it is based on a new MCGRP formulation which uses two-index variables also to model the link flow. Second, it takes advantage from all results of a lower bounding procedure. This procedure produces, besides excellent lower bounds, valid inequalities that are used to initialize a B&C scheme.

The remainder of the paper is organized as follows. In Section 2, a formal definition and the new two-index formulation of the MCGRP are given. In Section 3.1, we present the lower bounding formulation used in the exact approach illustrated in Section 3 in order to determine lower bounds and general cuts. Section 4 presents computational results. Final conclusions are drawn in Section 5.

2. Problem description and formulation

A formal definition of the MCGRP relies on a mixed graph $G = (V, E, A)$ with vertices set V, edges set E and arcs set A. Vertex $1 \in V$ represents the depot, at which a set K of homogeneous vehicles with capacity Q is based. The remaining vertices form the set $C = V \backslash \{1\}$. Every element $b \in V \cup E \cup A$ has a demand $q_b \geq 0$, those elements with strictly positive demand are *required*, meaning that they must be serviced exactly once. Required vertices are in $V_R = \{v \in C : q_v > 0\}$, required edges are in $E_R = \{e \in E : q_e > 0\}$, and required arcs are in $A_R = \{a \in A : q_a > 0\}$. In order to ensure feasibility, we assume that the demand q_r of each required element r does not exceed Q.

For notational ease, we speak of links when we want to refer to both edges and arcs in $E \cup A$. Any link can be *deadheaded*, i.e., traversed without being serviced, any number of times. The traversal of a link $\ell \in E \cup A$ results in a non-negative traversal cost c_ℓ. In the following, required elements are referred to as $r \in V_R \cup E_R \cup A_R$ when distinction is not essential.

The MCGRP is the problem of finding minimum-cost vehicle tours, each starting and ending at the depot, that together serve all required elements exactly once, and respect the vehicle capacity.

In order to state the MCGRP models, we introduce further notation used throughout the paper: Let S be a non-empty subset of vertices. We denote by $\delta^+(S)$ the set of arcs leaving S, by $\delta^-(S)$ the set of arcs entering S, by $\delta_R^+(S)$ the set of required arcs leaving S, by $\delta_R^-(S)$ the set of required arcs entering S, by $\delta(S)$ the set of edges with exactly one endpoint in S, and by $\delta_R(S)$ the set of required edges with exactly one endpoint in S. The associated link sets are $\delta^*(S) = \delta(S) \cup \delta^+(S) \cup \delta^-(S)$ and $\delta_R^*(S) = \delta_R(S) \cup \delta_R^+(S) \cup \delta_R^-(S)$. For the sake of brevity, singleton sets $S = \{i\}$ in the previous notation can be replaced by i so that, e.g., $\delta(i)$ stands for $\delta(\{i\})$. Finally, we denote by $V_R(S)$ the set of required vertices belonging to S, by $A_R(S)$ the set of required arcs with both endpoints in S, and by $E_R(S)$ the set of required edges with both endpoints in S.

We propose a new mathematical model based on variables with two indices, one for the respective vehicles $k \in K$ and the other for referring to an element of $V \cup E \cup A$. Let x_r^k be a binary variable equal to 1 if and only if the required element $r \in V_R \cup E_R \cup A_R$ is serviced by vehicle k. For a link $\ell \in E \cup A$ and a vehicle $k \in K$, let y_ℓ^k be a non-negative variable representing the number of deadheadings through ℓ by vehicle k. For a subset of required links $L \subseteq A_R \cup E_R$, we define $x^k(L) = \sum_{\ell \in L} x_\ell^k$, and for a subset of links $L \subseteq A \cup E$, we define $y^k(L) = \sum_{\ell \in L} y_\ell^k$. The two-index formulation for the MCGRP reads as follows:

$$\lambda^* = \min \sum_{k \in K} \sum_{\ell \in E \cup A} c_\ell y_\ell^k \quad \left(+ \sum_{\ell \in E_R \cup A_R} c_\ell \right) \tag{1a}$$

$$\sum_{k \in K} x_r^k = 1, \quad \forall\, r \in V_R \cup E_R \cup A_R \tag{1b}$$

$$\sum_{r \in V_R \cup E_R \cup A_R} q_r x_r^k \leq Q, \quad \forall\, k \in K \tag{1c}$$

$$x^k(\delta_R^*(i)) + y^k(\delta^*(i)) \equiv even, \quad \forall\, i \in V,\ k \in K \tag{1d}$$

$$x^k(\delta_R^-(S)) + y^k(\delta^-(S)) - x^k(\delta_R^+(S)) - y^k(\delta^+(S)) - x^k(\delta_R(S)) - y^k(\delta(S)) \leq 0, \quad \forall\, S \subset V,\ k \in K \tag{1e}$$