European Journal of Operational Research 243 (2015) 75–81

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

A single machine scheduling problem with two-dimensional vector packing constraints

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ARTICLE INFO

Article history:
Received 19 May 2014
Accepted 23 November 2014
Available online 9 December 2014

Keywords:
Scheduling
Two-dimensional vector packing
Recovering beam search
Matheuristics

ABSTRACT

We consider a scheduling problem where jobs consume a perishable resource stored in vials. It leads to a new scheduling problem, with two-dimensional jobs, one dimension for the duration and one dimension for the consumption. Jobs have to be finished before a given due date, and the objective is to schedule the jobs on a single machine so that the maximum lateness does not exceed a given threshold and the number of vials required for processing all the jobs is minimized. We propose a two-step approach embedding a Recovering Beam Search algorithm to get a good-quality initial solution reachable in short time and a more time consuming matheuristic algorithm. Computational experiments are performed on the benchmark instances available for the two-dimensional vector packing problem integrated with additional due dates to take into account the maximum lateness constraints. The computational results show very good performances of the proposed approach that remains effective also on the original two-dimensional vector packing instances without due dates where 7 new bounds are obtained.

1. Introduction

Scheduling problems are well studied in the literature since the 1950s and several books and recent review papers show the wide variety of problems considered today (Brucker, 2007; Hartmann & Briskorn, 2010; Pinedo, 2012; Ruiz & Vázquez-Rodriguez, 2010). In the same way, bin packing and vector packing problems have received a great attention in the literature and some survey papers are dedicated to these problems (De Carvalho, 2002; Lodi, Martello, & Vigo, 2002). In this paper, we introduce a new category of scheduling problems where vector packing constraints have to be considered together with the scheduling problem. These packing constraints are known in the literature as two-dimensional vector packing constraints (the problem may also be called two-constraint bin packing problem). We can refer to (Alves, de Carvalho, Clautiaux, & Rietz, 2014; Caprara & Toth, 2001; Garey, Graham, Johnson, & Yao, 1976; Spieksma, 1994) for resolution methods of this problem with m dimensions.

The origin of the problem comes from the production of chemotherapy drugs for cancer treatment by intravenous injection. In (Mazier, Billaut, & Tournamille, 2010), the authors describe this particular production environment and present a resolution method for a static or dynamic environment. In this production environment, the jobs to perform are called "*preparations*" and the raw materials are called *monoclonal antibodies* ("*products*" in the following). The monoclonal antibodies bind to specific cancer cells and induce an immunological response against the target cell. These products can be stored in vials for a long time before use, under specific storage temperatures and conditions. For some antibodies, freezing at −20 degree Celsius or −80 degree Celsius in small aliquots is the optimal storage condition. However, once a vial is opened or once the active agent has been mixed with a solute, it must be used before a given time limit, in order to keep intact the properties of the anticancer active agents. The maximum delay of use after opening depends on the agent and may vary between several hours to several days. In the meantime, the product has to be stored in a freezer or in a fridge for temperature and darkness reasons. If the time limit of use is exceeded, the monoclonal antibodies have lost their properties and they have to be destroyed according to a specific process. In other words, the time period between the starting time of the first preparation using a product in a vial and the completion time of the last preparation assigned to the same vial cannot be greater than the life time of the product. Furthermore, without loss of generality we assume that the total consumption associated to one vial cannot be greater than the total volume of the vial and we also assume

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http://dx.doi.org/10.1016/j.ejor.2014.11.036
0377-2217/

that two different vials cannot be assigned to the same preparation. We have here the two dimensions of the vector packing problem.

The cost of these products is very important. In Tours, the UBCO production center (described in Mazier et al., 2010) produces around 150 preparations per day. A preparation has an average cost of 400 euros, but it can reach 15,000 euros in certain cases. It is clear that the saving of these products may have an economic impact, absolutely not negligible. In the problem that we consider, one objective function is related to the waste of products, which we want to minimize. Notice that in such a context, the deadline for the use of the raw material becomes a variable of the problem, which is directly related to the production scheduling decisions: each time the life duration or the capacity of the vial is exceeded, a new vial is opened. Another version of this problem has been studied in Billaut (2011) and Billaut, Esquirol, and Tournamille (2011).

Furthermore, because each preparation has to be delivered to a patient for a given due date, another objective of the problem is to minimize the maximum lateness related to these due dates.

The same type of problem arises in concrete production because once prepared, the concrete has to be used within a given amount of time. Similarly, in food production, after the food is out of the freezer, it has to be used within a given time limit. Another possible application of this study arises for the resources management in the cloud environment (Padhy & Patra, 2013). For cloud providers, the problem is to allocate resources dynamically in the form of virtual machines to end users. Each task needs a virtual machine, i.e. a given quantity of RAM and of CPU. The tasks assigned to a resource cannot require more RAM and more CPU than the available quantity. Our problem, while different, presents also some similarities with parallel batch scheduling problems. But to the best of our knowledge, the problem that we consider in this paper has never been considered in the literature before.

The paper is organized as follows. In Section 2, the notations are introduced and the problem is formally defined. An MILP model is given. In Section 3, a recovering beam search and a matheuristic algorithm are proposed. These methods are tested on two dimensional packing benchmark instances, with due dates considerations and without due dates, for a comparison with methods dedicated to this problem. The results presented in Section 4 show that the proposed methods have very good performances, even without due date considerations. In this latter case, 7 new bounds are obtained with respect to the available literature. Section 5 presents the conclusions and some future research directions.

2. Problem statement and notations

We consider a simplified version of the problem of chemotherapy drugs production, assuming that there is only one machine and only one type of product (raw material). We have a set of n jobs to schedule on a single machine. To each job $J_j \in \{1, \ldots, n\}$ is associated a processing time p_j, a consumption b_j and a due date d_j. Without loss of generality, the jobs are supposed to be numbered in EDD order, i.e. $d_1 \le d_2 \le \cdots \le d_n$. The life duration of the product after opening is equal to T and the volume of one vial is equal to V. We assume without loss of generality that $p_j < T$ and $b_j < V, \forall j, 1 \le j \le n$. The number of vials is not limited but supposed to be bounded by n. We denote by C_j the completion time of J_j, L_j the lateness defined by $L_j = C_j - d_j$ and the maximum lateness is defined by $L_{\max} = \max_{1 \le j \le n} L_j$. We assume that the maximum lateness is bounded by a given value Q.

$$L_{\max} \le Q \tag{1}$$

We also assume that the remaining quantity of product in the last opened vial is lost at the end of the time horizon. Therefore, minimizing the quantity of lost product is equivalent to minimizing the number of vials that are opened. That is the reason why the problem is a mix between a scheduling problem and a two dimensional vector packing problem. Without due dates (or with extremely large due dates), the problem is exactly a two dimensional vector packing problem and thus clearly strongly NP-hard. With a big value of T and a big value of V, the problem is the classical single machine problem with the $L_{\max}$ minimization. In the following, we call a *bin*, the set of jobs performed with the same vial. It is well known from the literature (Spieksma, 1994) that a trivial lower bound on the number of bins is the following:

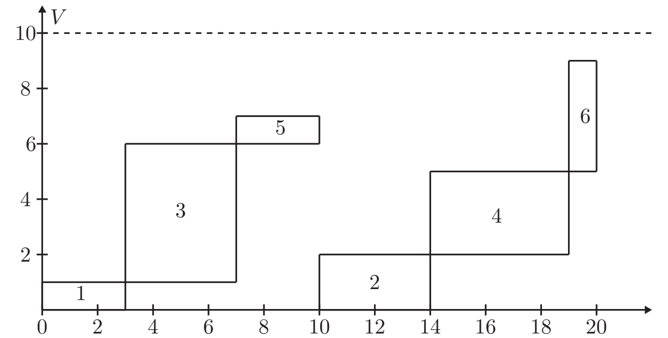

Fig. 1. Two-dimensional Gantt chart representation of schedule $(J_1, J_3, J_5, J_2, J_4, J_6)$.

$$\mathrm{LB} = \max\left(\left\lceil \sum_{j=1}^{n} p_j / T \right\rceil, \left\lceil \sum_{j=1}^{n} b_j / V \right\rceil\right). \tag{2}$$

We illustrate the problem by a numerical example.

2.1. Example

We consider a set of six jobs, $T = 10$ and $V = 10$.

j	1	2	3	4	5	6
p_j	3	4	4	5	3	1
b_j	1	2	5	3	1	4
d_j	7	9	11	13	14	16

Schedule $(J_1, J_3, J_5, J_2, J_4, J_6)$ is represented in Fig. 1. In this two dimensional Gantt chart, a job J_j is represented by a rectangle with the duration p_j on the x-axis and the consumption b_j on the y-axis. Jobs of the same bin are connected by the south-west corner of the rectangle. The first job of a bin is put on the x-axis. In Fig. 1, one can see that job J_1 is the first job of the bin composed by jobs $\{J_1, J_3, J_5\}$ and job J_2 is the first job of the bin composed by jobs $\{J_2, J_4, J_6\}$. Job J_2 cannot be included in the first bin because the duration of this bin would exceed T. The maximum lateness of this sequence is equal to $L_{\max} = \max(-4, 5, -4, 6, -4, 4) = 6$ and this schedule requires two bins. Notice that schedule $(J_1, J_2, J_3, J_4, J_5, J_6)$ is optimal for the $L_{\max}$, but requires three bins (see Fig. 2).

2.2. Dominance condition

Proposition 1. *In a bin, the jobs are scheduled in EDD order.*

This condition is clear because the order in a bin does not modify the number of bins that are used, and tends to improve the $L_{\max}$ value.

2.3. MILP formulation

We now propose an MILP formulation of the problem where the aim is to minimize the number of bins, assuming that the $L_{\max}$ value is bounded.