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Condition-based maintenance using the inverse Gaussian degradation model

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ABSTRACT

Condition-based maintenance has been proven effective in reducing unexpected failures with minimum operational costs. This study considers an optimal condition-based replacement policy with optimal inspection interval when the degradation conforms to an inverse Gaussian process with random effects. The random effects parameter is used to account for heterogeneities commonly observed among a product population. Its distribution is updated when more degradation observations are available. The observed degradation level together with the unit's age are used for the replacement decision. The structure of the optimal replacement policy is investigated in depth. We prove that the monotone control limit policy is optimal. We also provide numerical studies to validate our results and conduct sensitivity analysis of the model parameters on the optimal policy.

1. Introduction

This paper considers the condition-based maintenance (CBM) of a unit subject to a continuous-time stochastic degradation, and monitored through periodic inspections. CBM suggests regular inspections using modern sensor technology. Maintenance actions are then made based on the inspection of working conditions. Because CBM makes use of the in situ information, it most often outperforms the traditional age- and block-based maintenance policies. Therefore, the use of condition monitoring techniques and CBM has increased rapidly over recent years (see Jardine, Lin, & Banjevic, 2006; Wang, 2002, for reviews and examples).

The choice of the stochastic process that characterizes the physical deterioration, the so called prognostic models, will obviously affect the prediction of remaining useful life, and therefore influence the decision of the maintenance strategy and its economic performance. In the literature, many prognostic models have been proposed. In particular, when the health states are directly observable (Jardine et al., 2006) as in our study, they can be classified based on whether the degradation states are discrete or continuous. In the scenario with discrete states, Markov chain models (e.g. Bloch-Mercier, 2002; Chen, Chen, & Yuan, 2003; Xiang, Cassady, & Pohl, 2012; Yeh, 1997) are often adopted. After specifying the probability transition matrix among all states, these models can be used to calculate the time-to-failure distribution from any state. The Markov chain models are particularly useful when the degradation states of the unit cannot be precisely measured, and a rough category (e.g., good, moderate, weak) has to be adopted instead.

In recent decades, the fast development of sensing technologies enables accurate online measurements of the degradation levels. Under such circumstance, the degradation states are considered continuous as opposite to discrete. Consequently, stochastic models with continuous states are commonly used (e.g. Albin & Chao, 1992) to characterize the degradation process. Notably among them, the Wiener process, Gamma process, and the inverse Gaussian (IG) process, including their variants, attract significant attention because of their nice mathematical properties and clear physical interpretations (see Ye & Xie, 2014, for review). An attractive feature of these models is that they possess independent degradation increments. This property significantly simplifies the prediction of the remaining useful life given the historical degradation observations. As a result, it becomes much easier to determine the optimal maintenance policies. For example, optimal CBM policies based on the Wiener process degradation models have been extensively investigated (e.g. Elwany, Gebraeel, & Maillart, 2011; Guo, Wang, Guo, & Si, 2013; Hontelez, Burger, & Wijnmalen, 1996). However, since the Wiener process is not always monotone, it fails to model many degradations well such as crack growth and wears. For such monotone degradations, Gamma processes and IG processes are more suitable as they inherently impose the monotone constraints on the sample paths. CBM policies based on Gamma

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processes can be found in the literature as well. See Dieulle, Berenguer, Grall, and Roussignol (2003), Grall, Berenguer, and Dieulle (2002a), Liao, Elsayed, and Chan (2006) for examples and van Noortwijk (2009) for an excellent review. Nevertheless, different from Wiener process or Gamma process, which have been studied for a long time, IG process draws significant attention only recently. IG process was introduced by Wang and Xu (2010) to the reliability literature, and further investigated by Ye and Chen (2014). They had demonstrated significant advantages of IG process, including but not limited to, (i) clear physical interpretation for degradation caused by accumulated small damages; (ii) dual relationship with Wiener process, providing intuitive understanding; (iii) flexible ways to incorporate random effects and covariates to account for different types of heterogeneities; (iv) explicit analytical formulas for important quantities, such as PDF and CDF of remaining useful life distribution. However, despite the increasing attention on IG process (see e.g. Liu, Ma, Yang, & Zhao, 2014; Peng, Li, Yang, Huang, & Zuo, 2014; Wang & Xu, 2010; Ye & Chen, 2014; Ye, Chen, Tang, & Xie, 2014), there is scarce literature on CBM policies based on IG processes despite their useful applications.

While most of existing CBM policies assume that the prognostic models are fixed for all units across the population, this assumption is increasingly challenged. Because of diverse usage and environmental differences, the degradation characteristics of units from the same population are often different. As reported in many studies (e.g., Chen & Tsui, 2013; Gebraeel, Lawley, Li, & Ryan, 2005; Lawless & Crowder, 2004; Liao & Tian, 2013), heterogeneities often exist across the population, causing different degradation patterns. Many degradation models have been proposed to integrate the heterogeneities. Generally speaking, there are two approaches to take into account the heterogeneities. The first one is to use random effects models while the second is to impose a prior distribution on one or some of the model parameters. The crux of these two approaches is essentially the same, that is, some parameters are unit-specific and different across unit. The rest are common parameters shared by the entire population. As more degradation observations are collected, unit-specific parameter(s) can be estimated more precisely and can characterize the degradation process better. Unfortunately, while accounting for heterogeneity becomes common and has been well-recognized in degradation modeling and analysis, it also makes the optimal CBM policy difficult to find because the degradation process becomes non-stationary and age dependent.

In this paper, we develop an optimal inspection/replacement policy for units whose degradation can be modeled by an IG process. We incorporate the random effects in the IG process to model the unit-specific heterogeneity, and investigate the effects of such a heterogeneity on maintenance planning. The unit is inspected periodically. Upon each inspection, we make the decisions on whether a preventive replacement is needed. Failure is not self-announcing and can only be revealed through inspections. Operating in the failure state, the unit incurs a downtime cost. Our goal is to find the optimal *inspection interval* and *replacement policy* to minimize the total operational costs which include inspection cost, preventive/corrective replacement cost and downtime cost. We formulate the problem as a Markov decision process, and analyze the structural properties of the optimal policies. Our model shares some common features with the one proposed in Elwany et al. (2011), but it also differs from it in many aspects. The main differences are summarized as follows. First, the failure mechanism in our model is different. Elwany et al. (2011) used a Wiener process with linear drift to model the degradation, whose degradation paths are not strictly increasing. However, in many applications, the degradation is monotone. We consider the IG process with heterogeneity which is more suitable in such scenarios. Second, in Elwany et al. (2011) zero inspection cost is assumed, and the inspection interval becomes irrelevant to total operational cost. In practice, inspection costs might not be negligible, and it becomes crucial to determine the optimal inspection schedule to minimize the total cost. In our model, we explicitly consider the inspection costs and seek for optimal inspection interval in addition to the replacement policy. This formulation contains their model as a special case, and is expected to be more widely applicable. Third, we consider the downtime cost in our model. Since the failure is not self-announcing, the unit often incurs additional cost due to efficiency or quality loss when operating in the failure state. The downtime cost is especially alarming if the inspection interval is long, and consequently it becomes one deciding factor on the optimal inspection interval. Last but not least, we introduce a proof framework based on likelihood ratio ordering and associated properties. This technique is more generic, and can be applied more widely to other degradation models (e.g., Gamma processes with random shape parameters).

The remainder of the paper is organized as follows. Section 2 reviews the IG process and identifies important stochastic properties related to CBM. Section 3 formulates the CBM problem as a Markov decision process, and analyzed the structural information of the optimal policy in depth. Section 4 presents numerical studies to validate the theoretical results. It also provides sensitivity analysis to examine the effects of different parameters on the optimal maintenance decisions. Section 5 concludes the article with discussions. Some technical details and supplementary information are provided in Appendix A.

2. Inverse Gaussian process with heterogeneity

The inverse Gaussian (IG) process is recently proposed as a degradation model because it has many nice properties and it is flexible in dealing with covariates and random effects (Ye & Chen, 2014). Similar to the Gamma process, the IG process possesses monotone degradation paths. Mathematically, the IG process $\{Y_t, t \geq 0\}$ satisfies many important properties. It has independent increments, i.e., $Y_{t_1} - Y_{s_1}$ is independent of $Y_{t_2} - Y_{s_2}$ for $t_2 > s_2 \geq t_1 > s_1 \geq 0$. In addition, the increment $Y_t - Y_s$ follows an inverse Gaussian distribution $\mathcal{IG}(\mu[\Lambda(t) - \Lambda(s)], \eta[\Lambda(t) - \Lambda(s)]^2)$, where the transformed time scale function $\Lambda(t)$ is monotonically increasing with $\Lambda(0) = 0$, and $\mathcal{IG}(a, b), a, b > 0$ denotes the IG distribution. In addition to $\Lambda(t)$, the process parameters $\mu, \eta > 0$ control the degradation rate and degradation volatility.

In practice, different units often exhibit different degradation patterns due to a number of reasons, e.g., heterogeneous working conditions (Ye, Hong, & Xie, 2013) or variations in the raw materials. To accurately model and predict the degradation for each individual unit, it is important to account for such heterogeneity. In this article, we consider an IG process model with random effects where the degradation rate μ is assumed to be random. Different units have different realizations of μ, causing heterogeneities in their degradation rates. This model is called random-drift model throughout the paper. Prior to any degradation observations, μ^{-1} is typically assumed to follow a truncated normal distribution $\mathcal{TN}(\omega, \kappa^{-2}), \kappa > 0$ to exclude negative values, which has probability density function (PDF)

$$f(\mu^{-1}; \omega, \kappa^{-2}) = \frac{\kappa \cdot \phi[\kappa(\mu^{-1} - \omega)]}{1 - \Phi(-\kappa\omega)}, \quad \mu > 0, \tag{1}$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are PDF and cumulative distribution function (CDF) of the standard normal distribution, respectively.

Suppose the degradation of a unit is observed at times $t_j, j = 1, 2, \ldots, n$ with degradation levels $Y_j = Y_{t_j}$. Let $\Lambda_j = \Lambda(t_j)$. Given the observations $\mathbf{Y}_n = [Y_1, Y_2, \ldots, Y_n]$, μ^{-1} is still truncated normal. It is readily shown that $\langle \mu^{-1} | \mathbf{Y}_n \rangle \sim \mathcal{TN}\left(\tilde{\omega}_n, \tilde{\kappa}_n^{-2}\right)$ with $\tilde{\kappa}_n = \sqrt{\eta Y_n + \kappa^2}$ and $\tilde{\omega}_n = (\eta \Lambda_n + \omega\kappa^2)/\tilde{\kappa}_n^2$. The updated parameters of the truncated normal distribution only depend on Y_n and Λ_n. In particular, $\tilde{\kappa}_n$ increases as Y_n. When Y_n is large, $\tilde{\kappa}_n^{-2}$ becomes small and thus $\langle \mu^{-1} | \mathbf{Y}_n \rangle$ tends to degenerate to the true (but unobservable) value. Compared with the unconditional distribution of μ^{-1}, the conditional distribution $\langle \mu^{-1} | \mathbf{Y}_n \rangle$ is more accurate to predict the future degradation