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Decision Support

Multi-attribute sequential decision problem with optimizing and satisficing attributes

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ABSTRACT

We deal with the multi-attribute decision problem with sequentially presented decision alternatives. Our decision model is based on the assumption that the decision-maker has a major attribute that must be "optimized" and minor attributes that must be "satisficed". In the vendor selection problem, for example, the product price could be the major factor that should be optimized, while the product quality and delivery time could be the minor factors that should satisfy certain aspiration levels. We first derive the optimal selection strategy for the discrete-time case in which one alternative is presented at each time period. The discrete-time model is then extended to the continuous-time case in which alternatives are presented sequentially at random times. A numerical example is used to analyze the effects of the satisficing condition and the uncertainty on the optimal selection strategy.

1. Introduction

In many decision problems, not all decision alternatives are available to the decision-maker (DM) simultaneously. Instead, the DM evaluates only one alternative at a time, and decides whether to make a final choice or continue searching for better alternatives (Korhonen, Moskowitz, Salminen, & Wallenius, 1993). Consider, for example, the problem of hiring an employee or buying a house in the open market. Several decision alternatives (or choices) are presented to the DM randomly and sequentially over time. After evaluating a choice currently available, the DM may either select it and terminate the search process, or reject it and continue the uncertain search process.

In some situations, any choices that have been rejected cannot be recalled at a later stage. Therefore, if the search process is terminated too early, choices superior to the selected choice may not have been presented yet; if the selection is made too late, the superior choices may have been rejected earlier in the search process. In such a case, the DM's goal is to derive the optimal stopping rule that maximizes the probability of selecting the "best choice" (Chun, 1999).

In many complex decision problems, each choice is evaluated in terms of multiple conflicting attributes. We may simply assume that only one of those attributes is a major factor and other attributes are minor ones that can be ignored. Then, the multi-attribute sequential decision problem is simply reduced to a single-attribute decision problem with only one major attribute. In the house *selling* problem, for example, the most important attribute is the offer from a potential buyer (Chun, Plante, & Schneider, 2002). From the seller's point of view, the objective is to find the highest offer in terms of dollar value within a limited time period.

In the house *buying* problem, on the other hand, each house is compared in terms of multiple attributes (or dimensions), such as the asking price, size, age of the house, and so forth. From the buyer's point of view, buying a house is presented as a multi-attribute sequential decision problem. The job search problem is another example of the multi-attribute sequential problem. The job offers are usually compared based on the starting salaries, fringe benefits, locations, future growths, and so forth (Bearden, Murphy, & Rapoport, 2005). In the vendor selection problem in supply chain management, the most popular evaluating criteria are product price, quality, and delivery time. The marriage problem (or the bachelor's dilemma) also involves many conflicting objectives such as the appearance, intelligence, personality, financial security, and so on.

The single attribute sequential decision problem is also known as the secretary problem, marriage problem, job search problem, parking spot problem, or asset-selling problem, with each of them using different assumptions. Since its introduction in the early 1960s, this particular field of study has experienced rapid growth, and its applications extend to a wide variety of managerial decision problems. Readers who are more interested in various types of single-attribute sequential decision problems are referred to many excellent review papers, including Bearden and Rapoport (2005), Ferguson (1989), and Freeman (1983).

For the non-sequential version of multi-attribute decision problems, many decision models, such as analytic hierarchy process

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(AHP), genetic algorithms, simple multi-attribute rating techniques (SMART), fuzzy set theory, and data envelopment analysis (DEA), have been proposed (Ho, Xu, & Dey, 2010). The goal programming is one of the most popular methods in treating multi-attribute decision problems (Aouni & Kettani, 2001; Tamiz, Jones, & Romero, 1998). The goal programming and its variants have been successfully applied to a wide variety of multiple objective/criteria decision problems (Abdelaziz, 2007; Aouni, Colapinto, & La Torre, 2014; Köhn, 2011).

Only recently has serious consideration been given to the sequential decision problem with multiple attributes, which has inherent difficulties in the formulation of models. The first difficulty is that there might not be a single choice that "globally" dominates all of the other choices in terms of all of the multiple attributes (Korhonen, Moskowitz, & Wallenius, 1986). The second difficulty is that it is also not realistic to assume the DM's multi-attribute utility function to be known explicitly (Korhonen & Wallenius, 1986). Finally, the multiple attributes are statistically inter-dependent with each other. Despite these difficulties, some authors have assumed that the DM has a given utility function of multiple attributes with known parameter values or that the DM can somehow rank-order the choices.

Bearden and Connolly (2007) considered two types of search strategies for the multi-attribute sequential selection problem. They assume that the multi-attribute utility function and its parameter values are given, and the value of a choice is simply the sum of its attribute values. Recently, Smith, Lim, and Bearden (2007) and Lim, Bearden, and Smith (2006) considered a similar multi-attribute sequential selection problem with a search cost. They also assume that the multi-attribute value function is a separable function, and that the DM's objective is to maximize the expected payoff. In the rank-based sequential selection problem, Bearden and Rapoport (2005) assumed that attributes are statistically independent with each other, and that the value of a choice is the sum of its rankings with each attribute.

In the paper, we overcome the inherent difficulty by dividing multiple attributes into two groups – one "major" and several "minor" attributes. Dividing or prioritizing multiple attributes is not uncommon in multi-objective decision analysis. In pre-emptive goal programming, for example, the decision maker prioritizes his or her goals into different priority levels (Jones & Jimenez, 2013). The idea behind the preemptive goal programming approach is that lower priority level goals should not be attained at the expense of higher priority goals. As a result, some of the goals satisfy their aspiration levels, but other goals may not achieve their aspiration levels perfectly.

We then assume that a major attribute should be "optimized", whereas minor attributes should be "satisficed". (The term *satisficing* is a portmanteau of *satisfy* and *suffice*.) In the house buying problem, for example, a buyer's goal is to find the least expensive house among the ones that have at least 2000 square feet of living area and are less than 10 years old. In such a case, the asking price is the major attribute (or goal) that should be optimized, and the size and the age of a house are the minor attributes (or constraints) that should be satisfied.

The assumption significantly simplifies the search process and the computations involved in executing it. Any choices that satisfy the minimum requirements of the minor attributes are called "acceptable". Among the acceptable choices, the "best choice" is defined as the one that has the best value on its major attribute. In the paper, we propose a rank-based sequential selection strategy that maximizes the probability of selecting the best choice.

The idea of satisficing and optimizing attributes is adopted from the two cognitive styles suggested by Simon (1955, 1959). A satisficing conception of rationality will permit the "suitable means" to be good enough; a maximizing conception will require the "suitable means" to be the best (Byron, 1998). Thus, "satisfiers" usually set an aspiration level and simply try to find any choice that reaches or exceeds that level. On the other hand, "maximizers" try to make an optimal decision among all feasible choices.

These two distinct approaches to human decision-making have been widely studied in economics and behavioral science (Bearden & Connolly, 2008; Byron, 1998; Sen, 1997). The two approaches have also been applied in the field of operational research. Chance-constrained programming (Charnes & Cooper, 1963) and goal programming (Tamiz et al., 1998) in multi-criteria decision analysis are well-known examples of the satisficing approach. As far as I know, there has not been a research effort devoted to considering the "optimizing" and "satisficing" attributes in the context of the sequential decision analysis with multiple attributes.

The paper is organized as follows: In Section 2, we formally define several terms and introduce notations that will be used throughout the paper. The classical "discrete-time" model with a known number of choices is formulated in Section 3, which is modified in Section 4 in order to consider the uncertain availability of a choice at each stage. In Section 5, we consider the "continuous-time" model in which choices are presented at irregular intervals. Section 6 is devoted to the sensitivity analysis of discrete-time and continuous-time models, followed by concluding remarks in Section 7.

2. Preliminaries

In a sequential decision problem, we assume that n choices will be presented to the DM sequentially in a random order. The total number of choices, n, is a known constant in the discrete-time case. In the continuous-time case, the inter-arrival time between two successive choices follows a continuous Markov process, and thus the total number of choices is unknown. In the continuous-time case, we assume that the DM must make a decision before a given due date.

Each choice will be evaluated based on multiple attributes. One of them is identified as a "major" attribute, and the rest of them are regarded as "minor" attributes. Without loss of generality, we assume that the DM's utility is *increasing* in each attribute (i.e., the higher is the better) (Korhonen & Wallenius, 1986). A choice is said to be "acceptable" if all of its minor attributes satisfy pre-specified aspiration levels. Among the acceptable choices identified up to the current stage, the choice with the highest value on its major attribute is referred to as the "relatively best choice" or the "candidate". The "absolutely best choice" (or simple the "best choice" in short) is the candidate with the highest value on its major attribute from all the candidates.

In accordance to the dynamic programming formulation of a sequential decision problem, we define the stage, state, and decision as follows: When m more choices will be presented for further consideration, the DM is said to be at the mth "stage" in the discrete-time search process. In the continuous-time case, the "stage" is defined as the remaining time t until the due date. The "state" at each stage is simply represented as a zero-one binary variable, 1 representing that the choice currently under consideration is a candidate, and 0 representing that the current choice is not a candidate.

If the state is 1 at the current stage, the DM's "decision" is either (1) to select the candidate currently available and stop the search process or (2) to reject the current candidate and wait for another choice. If the state is 0 at the current stage, the current choice is not a candidate and the DM cannot win with that choice. Thus, the DM must continue the search process at a stage if its state is 0. The "optimal decision" at a given stage is the one that maximizes the probability of winning the game or selecting the absolutely best candidate.

Consider, for example, a two-attribute sequential decision problem with a major attribute X and a minor attribute Y. As shown in Fig. 1, suppose that 9 choices will be presented one at a time. Let s be the satisficing condition on the minor attribute Y; any choices with its value $y_i < s$ are not acceptable. Thus, the first, fourth, and eighth choices in Fig. 1 are unacceptable and should be eliminated from further considerations.