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Decision Support

Optimal savings management for individuals with defined contribution pension plans

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ABSTRACT

The paper provides some guidelines to individuals with defined contribution (DC) pension plans on how to manage pension savings both before and after retirement. We argue that decisions regarding investment, annuity payments, and the size of death sum should not only depend on the individual's age (or time left to retirement), nor should they solely depend on the risk preferences, but should also capture: (1) economical characteristics—such as current value on the pension savings account, expected pension contributions (mandatory and voluntary), and expected income after retirement (e.g. retirement state pension), and (2) personal characteristics—such as risk aversion, lifetime expectancy, preferable payout profile, bequest motive, and preferences on portfolio composition. Specifically, the decisions are optimal under the expected CRRA utility function and are subject to the constraints characterizing the individual.

The problem is solved via a model that combines two optimization approaches: stochastic optimal control and multi-stage stochastic programming. The first method is common in financial and actuarial literature, but produces theoretical results. However, the latter, which is characteristic for operations research, has practical applications. We present the operations research methods which have potential to stimulate new thinking and add to actuarial practice.

1. Introduction

Recent years have seen a decided worldwide shift from defined benefits (DB) pension plans toward defined contribution (DC). The number of participants in DC plans is quickly expanding because these plans are not only easier and cheaper to administer, but also more transparent and more flexible. Furthermore they can better capture the individual's needs. However, DC plans also pose some challenges, namely, the participants often do not know how to manage their saving and investment decisions.

In some countries, such as the United States, most DC decisions are made by the individual with little advice from the employer. In contrast, in countries such as Denmark, the sponsoring organizations, including life insurers, suggest a dynamic investment strategy suitable to the individual's age and risk preferences. Individuals in most of the countries also have to decide on how to spend the amount accumulated on their pension savings account. Should they follow a certain withdrawal rate rule, or should they purchase annuities that will provide with regular payments during retirement? This task is not easy, especially when life insurers offer a wide variety of annuity products (e.g. fixed or variable, deferred or immediate, term or whole-life). How can the individuals know, which product is best for them?

There is one more decision they have to keep in mind. Namely, what to do with the savings in case of their death? Do they want to bequeath the savings to their heirs, or maybe purchase an annuity product combined with a life insurance policy? What level of death sum should they choose?

We argue that aforementioned decisions should differ for each individual and should account for the following factors: (1) economical characteristics – such as current value on the pension savings account, expected pension contributions (mandatory and voluntary), and expected income after retirement (e.g. retirement state pension), and (2) personal characteristics – such as risk aversion, lifetime expectancy, preferable payout profile, bequest motive, and preferences on portfolio composition.

To help the individuals manage the savings and investment decisions we build an optimization-based financial planning model. Because such a model can be complicated and difficult to solve, we propose to combine two popular methodologies: multi-period stochastic programming (MSP) and stochastic optimal control (SOC), also referred to continuous-time and state dependent dynamic

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programming. The latter method is common in financial and actuarial literature, and its main advantage is the analytical form of the optimal solution, which is easy to understand and implement. See, for example, Yaari (1965), Samuelson (1969), Merton (1969, 1971), Richard (1975) and Campbell and Viceira (2002), for optimal decisions regarding investment, consumption and sum insured. However, the main drawback of this approach is that the explicit solution in many cases does not exist.

On the contrary, MSP, which is characteristic for operations research, has practical application and complement SOC approach, especially in terms of adding realistic constraints and modeling more complicated processes. In stochastic programming approach we model the possible outcomes for the uncertainties in a scenario tree, and numerically compute the optimal solution at each node of the tree. See, for example, Carino, Myers, and Ziemba (1998) and Carino and Ziemba (1998), who formulate a financial planning model for one of the biggest Japanese property and casualty insurer, Mulvey, Simsek, and Pauling (2003), who present a multi-period stochastic network model for integrating corporate financial and pension planning, and Mulvey, Simsek, Zhang, Fabozzi, and Pauling (2008) who expand this work by adding the borrowing decisions. The applications of MSP to individual asset-liability management can be found, for example, in Ziemba and Mulvey (1998), Kim, Mulvey, Simsek, and Kim (2012) and Konicz and Mulvey (2013). However, the main drawback of this optimization method is the limited ability to handle many periods under enough uncertainty about future asset returns and human lifetime. Especially, modeling the entire lifetime of an individual is challenging in terms of computational tractability.

To benefit from both optimization approaches and to avoid the aforementioned drawbacks, we combine multi-stage stochastic programming and stochastic optimal control into one mathematical framework. We solve the problem using MSP approach up to some horizon T, and to ensure that the model accounts for the entire lifetime of an individual, we insert the *end effect* in the objective function of MSP. The end effect is determined by the optimal value function calculated explicitly via SOC technique. This function covers the period from the horizon T to the individual's death. Combining these two optimization approaches is new and has only been investigated in Geyer, Hanke, and Weissensteiner (2009) and Konicz, Pisinger, Rasmussen, and Steffensen (2014). The presented MSP framework can be posed and solved with reasonable efficiency, while providing reliable and robust insights. These policy rules can further be implemented using Monte Carlo simulations, which are simpler and more likely to be employed in practice than complicated stochastic models, see, e.g., Mulvey et al. (2008).

The paper is organized as follows. Section 2 describes the economical and personal characteristics that we take into account when advising on how to manage the pension savings. Section 3 presents the financial planning model. Section 4 explains the intuition behind the optimal solution obtained from MSP model. Section 5 includes numerical examples illustrating the application of the model for different individuals. Section 6 concludes. Finally, Appendix A introduces multi-stage stochastic programs and Appendix B presents details of the explicit solution derived via SOC approach.

2. Economical and personal characteristics

We argue that management of savings in DC pension plan should account for economical and personal characteristics, and it should be tailored to a customer. Our model takes into account the following factors.

2.1. Economical characteristics

2.1.1. Current value on the savings account

The value of the individual's account, X_t, develops according to the initial savings x_0, contributed premiums, capital gains including dividends, insurance coverage, accredited survival credit and the benefits paid after retirement—all these elements are described below.

2.1.2. Premiums

Until retirement the individual contributes to the savings account. The premiums P_t^{tot} consist of a fixed percentage p^{fixed} of the labor income l_t, which is in many countries mandatory and decided by the employer, and the additional voluntary contributions, $p^{\text{vol}}\, l_t$. The latter may be of interest of an individual who wishes to increase the future benefits.

$$P_t^{\text{tot}} = (p^{\text{fixed}} + p^{\text{vol}})l_t, \quad p^{\text{fixed}} \in [0, 1], \quad p^{\text{vol}} \in [0, 1 - p^{\text{fixed}}].$$

The labor income l_t is deterministic and increases with a salary growth rate y_l, $l_t = l_0 e^{y_l t}$, where l_0 is the level of the labor income at the current time t_0. Both the premiums and the labor income are positive only until retirement, $t < T_R$; otherwise 0.

2.1.3. State retirement pension

After retirement the individual has no other income than state retirement pension, b_t^{state}. This income is typically financed on a pay-as-you-go basis from general tax revenues, and ensures a basic standard of living for old age. It often depends on the level of the individual's income before retirement, but not on the income from the DC plan. We assume that the state retirement pension consists of the life long, yearly adjusted payments.

2.2. Personal preferences

2.2.1. Risk aversion

The individual is risk averse and obtains a utility u from the total pension benefits B_t^{tot} and from leaving money upon death to the heirs, Beq_t. The utility function is characterized by a constant relative risk aversion (CRRA) $1 - \gamma$ and time dependent weights w_t:

$$u(t, B_t^{\text{tot}}) = \frac{1}{\gamma} w_t^{1-\gamma} (B_t^{\text{tot}})^{\gamma}, \quad u(t, \text{Beq}_t) = \frac{1}{\gamma} w_t^{1-\gamma} (\text{Beq}_t)^{\gamma},$$

where $\gamma \in (-\infty, 1) \setminus \{0\}$, whereas $\gamma = 0$ implies the logarithmic utility. Time dependent weights w_t include an impatience weighted interest factor ρ,

$$w_t = e^{-\rho t/(1-\gamma)},$$

which allows the individual to specify how important the benefits and the death sum are at the present moment relatively to how important these payments would be in the future. Thus, $\rho = 0$ implies that the current and future payments are equally important for the individual, and $\rho > 0$ reflects that the weight on the future payments decreases exponentially with time.

2.2.2. Lifetime expectancy

The individual has uncertain lifetime, which we model with two kinds of mortality rates: μ_t and ν_t. The first function denotes the subjective mortality rate and reflects the individual's expectation about her mortality rate. The lifetime expectancy is either based on the individual's lifestyle and health status or simply on the individual's opinion. For example, does she live a healthy lifestyle and therefore expect to live longer than others? Is she a regular smoker or maybe seriously ill? Does she expect to live longer than an average individual despite a smoking habit? The choice of the subjective mortality rate μ_t affects the decisions regarding the payout profile as well as the decision about purchasing life insurance.

The second function, ν_t, also referred to pricing mortality, is used by life insurers for calculating the price of their life contingent products. Especially in European countries, due to legislation, both the survival credit and the price for life insurance are calculated under unisex criteria, and the individual is not even subject to health screening, see Rocha, Vittas, and Rudolph (2010). A person with a cancer disease,