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Decision Support

Generalized ordered weighted utility averaging-hyperbolic absolute risk aversion operators and their applications to group decision-making

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ABSTRACT

This paper develops a new class of aggregation operator based on utility function, which introduces the risk attitude of decision makers (DMs) in the aggregation process. First, under the general framework of utility function, we provide a new operator called the generalized ordered weighted utility averaging (GOWUA) operator, and study its properties which are suitable for any utility function. Then, under the hyperbolic absolute risk aversion (HARA) utility function, we propose another new operator named as the generalized ordered weighted utility averaging-hyperbolic absolute risk aversion (GOWUA-HARA) operator, and further investigate its families including a wide range of aggregation operators. To determine the GOWUA-HARA operator weights, we put forward an orness measure of the GOWUA-HARA operator and analyze its properties. Considering that different DMs may have different opinions toward decision-making and their opinions can be characterized by different orness measures, we construct a new optimization model to determine the optimal weights which can aggregate all the individual sets of weights into an overall set of weights. Furthermore, based on the GOWUA-HARA operator, a method for the multiple attribute group decision-making (MAGDM) is developed. Finally, an example is given to illustrate the application of the GOWUA-HARA operator to the MAGDM.

1. Introduction

Multiple attribute group decision-making (MAGDM) considers the problem of evaluating or selecting alternatives that are associated with incommensurate and conflicting criteria by a cooperative group, known as the group decision-making (Vahdani et al., 2013). To choose a desirable alternative, decision makers (DMs) often present their preference information which needs to be aggregated via some proper approaches. There are many methods for aggregating information (Canós & Liern, 2008; Chiclana, Herrera-Viedma, Herrera, & Alonso, 2007; Dong, Xu, Li, & Feng, 2010; Fullér & Majlender, 2003a, 2003b; Liu, 2013; Llamazares, 2004; Maes, Saminger, & De Baets, 2007; Merigó, 2008; Merigó & Casanovas, 2010a, 2010b; Merigó, Casanovas, & Yang, 2014; Merigó, Casanovas, & Zeng, 2014; Merigó & Gil-Lafuente, 2010, 2011, 2013; Merigó & Yager, 2013; Mesiar, 2007; Ribeiro & Pereira, 2003; Xu, 2004, 2006a, 2006b; Xu, Yang, & Wang, 2006; Yager, 1988, 2004, 2010; Yager & Filev, 1994, 1999; Yang, 2001; Yang, Yang, Liu, & Li, 2013; Zeng, Merigó, & Su, 2013; Zhang, 2013; Zhou & Chen, 2014). One of the most popular methods for aggregating decision-making information is the ordered weighted averaging (OWA) operator developed by Yager (1988). It provides a general class of parametric aggregation operators and has shown to be useful for studying many different kinds of aggregation problems. Up to now, the OWA operator has been used in a wide range of applications (Merigó & Gil-Lafuente, 2010, 2011; Yager, 2010).

Motivated by the OWA operator, an extension of the OWA operator is the generalized OWA (GOWA) operator, which combines the OWA operator with the generalized mean operator (Yager, 2004). It generalizes a wide range of aggregation operators such as the OWA operator, the ordered weighted geometric averaging (OWGA) operator (Xu & Da, 2002a), the ordered weighted harmonic averaging (OWHA) operator (Yager, 2004). Based on the optimization theory, Zhou and Chen (2010) presented the generalized ordered weighted logarithm averaging (GOWLA) operator, which is an extension of the OWGA operator. Other extension of the OWA operator can be found in literature (Merigó, 2008; Yager & Filev, 1999). However, the above aggregation operators only focus on using the mean to eliminate the difference, and do not consider the DMs' risk attitude in the aggregation process.

Another important issue of applying the OWA operator for MAGDM is how to determine the associated weights. Many researchers have focused on this issue and developed some useful approaches to obtaining the OWA weights. For example, O'Hagan (1988) suggested a maximum entropy approach to obtaining OWA operator

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weights under a given orness measure. Yager (1993) suggested an interesting way to compute the weights of the OWA operator using linguistic quantifiers. Fullér and Majlender (2003a) proposed an analytic approach for obtaining maximal entropy OWA operator weights under a given orness measure. Wang and Parkan (2005) proposed a minimax disparity approach for obtaining the OWA operator weights under a given orness measure. Majlender (2005) developed a maximal Rényi entropy method for generating a parametric class of the OWA operators and the maximal Rényi entropy OWA weights. Wang, Luo, and Liu (2007) constructed the chi-square (χ^2) model for determining the OWA operator weights under a given orness measure. Other extension approaches to determining the OWA operator weights can be found in literature (Ahn, 2010; Amin & Emrouznejad, 2010; Filev & Yager, 1998; Merigó, 2008; Sang & Liu, 2014; Xu, 2006a, 2006b; Xu & DA, 2002b, 2003; Yager, 1988, 2009a, 2009b). The methods mentioned above assume that any individual weighting vector is equal to the optimal aggregated weighting vector, and correspondingly there is only one orness measure to characterize the DMs' attitude toward decision-making. As a result, there is only one set of the OWA operator weights to be generated. However, this is not consistent with the real situation. In fact, multiple DMs may join in decision-making process to reach a holistic opinion that reflects a collective view of all the participants. In the decision-making process, different DMs may have different orness measures, and therefore the corresponding OWA operator weights may also be different. So it is necessary to introduce a new method to aggregate all the participants' preference in MAGDM.

This paper aims to develop a new class of aggregation operator based on utility function, which incorporates the risk attitude of DMs in the aggregation process. Under the general framework of utility function and based on an optimal deviation model, we firstly provide a new operator called the generalized ordered weighted utility averaging (GOWUA) operator, and then by studying its properties we find that it is commutative, monotonic, bounded and idempotent. These properties are suitable for any utility function. Furthermore, we focus on the hyperbolic absolute risk aversion (HARA) utility function, which is rather rich, e.g., by suitable adjustments of the parameters, power utility function and exponential utility function can be obtained respectively. Under the HARA utility, we propose another new operator called the generalized ordered weighted utility averaging-hyperbolic absolute risk aversion (GOWUA-HARA) operator, and study its families which contain a wide range of aggregation operators such as the OWGA operator, OWA operator, OWHA operator, GOWA operator, maximum operator, minimum operator. The main advantage of the GOWUA-HARA operator is that it cannot only reflect the DMs' risk attitude toward the aggregation information, but also provide a very general formulation including a wide range of aggregation operators.

In order to determine the weights of the GOWUA-HARA operator, we put forward an orness measure of the GOWUA-HARA operator, which is an extension of the orness measure of the GOWA operator presented by Yager (2004). We further investigate some properties associated with this orness measure. Noting that different DMs may have different perspectives toward decision-making, which can be characterized by different orness measures, this situation leads to different sets of the GOWUA-HARA operator weights corresponding to different orness measures. We then construct a new nonlinear optimization model to determine the optimal weighting vector of the GOWUA-HARA operator which can aggregate all the individual sets of weights into an overall set of weights. The main advantage of the nonlinear model is that it cannot only minimize the differences between the orness measures provided by each DM and the comprehensive orness measure corresponding to an optimal weighting vector, but also produce as equally important weights as possible.

Furthermore, based on the GOWUA-HARA operator, a new approach for MAGDM is developed. This approach is also effectively applicable to different group decision-making problems such as engineering management and financial management. In the end, we provide an application of the new approach for MAGDM in an example of the investment selection.

The rest of the paper is organized as follows. Section 2 reviews the OWA, OWGA and GOWA operators and introduces an HARA utility function. Section 3 presents a GOWUA operator and analyzes its properties. Especially, we provide a GOWUA-HARA operator and identify its families. Section 4 proposes an orness measure of the GOWUA-HARA operator and discusses its properties. In particular, we further construct a nonlinear model for determining the optimal weights which can aggregate each DM's opinion. Section 5 develops an approach for MAGDM under the GOWUA-HARA operator. An illustrative example is provided in Section 6 and the conclusions are drawn in Section 7.

2. Preliminaries

This section briefly reviews the OWA, OWGA and GOWA operators and introduces an HARA utility function which later will be used to develop a new aggregation operator in this paper.

2.1. The OWA operator

The ordered weighted averaging (OWA) operator was presented by Yager (1988), which can be defined as follows:

Definition 1. (Yager, 1988). A mapping OWA: $R^n \to R$ is called an ordered weighted aggregation (OWA) operator of dimension n if

$$\mathrm{OWA}(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} w_i y_i, \tag{1}$$

where w_i is a weight satisfying $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, y_i is the ith largest of the x_j.

2.2. The OWGA operator

Xu and Da (2002a) provided the ordered weighted geometric averaging (OWGA) operator, which can be defined as follows:

Definition 2. (Xu & Da, 2002a). An ordered weighted geometric averaging (OWGA) operator is a mapping OWGA: $R^{+n} \to R^+$ that has a weighting vector $W = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, such that

$$\mathrm{OWGA}(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} y_i^{w_i}, \tag{2}$$

where y_i is the ith largest of the x_j.

2.3. The GOWA operator

Yager (2004) developed the generalized ordered weighted averaging (GOWA) operator, which is defined as follows:

Definition 3. (Yager, 2004). A generalized ordered weighted aggregation (GOWA) operator of dimension n is a mapping GOWA: $R^{+n} \to R^+$ that has a weighting vector $W = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ with $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$, such that

$$\mathrm{GOWA}\,(x_1, x_2, \ldots, x_n) = \left(\sum_{i=1}^{n} w_i {y_i}^{\lambda}\right)^{1/\lambda}, \tag{3}$$

where y_i is the ith largest of x_j, and λ is a parameter such that $\lambda \in (-\infty, 0) \cup (0, +\infty)$.

By taking different values of the parameter λ in the GOWA operator, a group of particular cases can be derived. For example, the