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Discrete Optimization

Incremental network design with shortest paths

Matthew Baxter [a], Tarek Elgindy [a], Andreas T. Ernst [a], Thomas Kalinowski [b], Martin W.P. Savelsbergh [b,*]

[a] CSIRO Mathematics Informatics and Statistics, CSIRO, Australia
[b] University of Newcastle, Australia

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ABSTRACT

We introduce a class of incremental network design problems focused on investigating the optimal choice and timing of network expansions. We concentrate on an incremental network design problem with shortest paths. We investigate structural properties of optimal solutions, show that the simplest variant is NP-hard, analyze the worst-case performance of natural greedy heuristics, derive a 4-approximation algorithm, and conduct a small computational study.

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1. Introduction

Consider a network optimization problem, e.g., a shortest path problem, a maximum flow problem, or a traveling salesman problem. Next, assume that this optimization problem has to be solved in a number of consecutive time periods and that in each time period the value of an optimal solution is incurred, e.g., the cost of an s—t path, the value of an s—t flow, or the cost of a TSP tour. Let the objective be to minimize or maximize the total (cumulative) cost or value over the planning horizon. At this point, that simply means solving the network optimization problem and multiplying the value of an optimal solution with the number of time periods in the planning horizon. It becomes more interesting when a budget is available in each time period to expand the network, i.e., to build additional links. Expanding the network may improve the cost or value of an optimal solution to the network optimization problem in future time periods and thus may improve the total cost or value over the planning horizon. However, deciding which links to build and the sequence in which to build them is nontrivial. In part, because in some situations the benefits of building a link will only materialize when other links have been built as well, e.g., adding a single link to the network does not lead to a shorter TSP tour, but adding two links to the network does.

We introduce a class of incremental network design problems that focuses on the *optimal choice and timing of network expansions* given that these network expansions impact the value of a solution to an optimization problem that is solved on the network in each of the periods of the planning horizon.

We are primarily interested in establishing the complexity of incremental network design problems, and, in case they are NP-hard, in their approximability, i.e., establishing whether a ρ-approximation algorithm exists, where an algorithm achieves an approximation ratio $\rho \geqslant 1$ for a minimization problem if, for every instance, it produces a solution of value at most ρv^{opt} with v^{opt} the value of an optimal solution. Therefore, we focus on what appears to be one of the most basic incremental network design problems, namely the incremental network design problem with shortest paths.

We investigate structural properties of optimal solutions, show that even the simplest variant is NP-hard, establish a class of instances that can be solved in polynomial time, analyze the worst-case performance of natural greedy heuristics, derive a 4-approximation algorithm, and conduct a small computational study.

Even though single-stage or single-period network design problems have been studied extensively by the operations research community, multi-stage or multi-period network design problems, which occur just as often in practice, have received much less attention. We hope that our investigation demonstrates that multi-period network design problems present interesting challenges and can produce intriguing and surprising results.

The remainder of the paper is organized as follows. In Section 2, we introduce the class of incremental network design problems. In Section 4, we introduce the incremental network design problem

* Corresponding author. Tel: +61 2 4921 5534; fax: +61 2 4921 6898.
E-mail address: martin.savelsbergh@gmail.com (M.W.P. Savelsbergh).

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with shortest paths. In Sections 5–7, we analyze the complexity of the incremental network design problem with shortest paths, and we explore greedy heuristics and design an approximation algorithm, respectively. In Section 8, we introduce an integer programming formulation and present the results of a small computational study. Finally, in Section 9, we discuss possible extensions and future research.

2. A class of incremental network design problems

Incremental network design problems have two characteristic features: a design feature, since we are deciding which arcs will be part of a network, and a multi-period feature, since the ultimate network design is built over a number of time periods.

The general structure of an incremental network design problem is as follows. We are given a network $D = (N, A)$ with node set N and arc set $A = A_e \cup A_p$, where A_e contains *existing* arcs and A_p contains *potential* arcs. Each arc $a \in A$ has a capacity C_a. Let T be the planning horizon. A budget B^t is available in every time period $t \in \{1, \dots, T\}$. The budget can be used to build potential arcs $a \in A_p$, which will be available for use in the following period. For each potential arc $a \in A_p$, there is an associated build-cost $c_a \leqslant B$. Let y_a^t be a 0–1 variable indicating whether arc $a \in A_p$ has been built in or before time period t, with all potential arcs initially unbuilt ($y_a^0 = 0$). Thus, $y_a^t - y_a^{t-1} = 1$ indicates that arc a is built in time period t and can be utilized in period $t+1$. In every time period, a network optimization problem P has to be solved over the usable arcs in time period t, i.e., the existing arcs and the potential arcs that have been built before time period t. Let x_a^t represent the flow on arc $a \in A$ in time period $t \in \{1, \dots, T\}$ in an optimal solution to the network optimization problem. Let $F(P)$ define the "structure" of feasible solutions to the network optimization problem, i.e., the set of constraints imposed on the flow variables (that it has to be an s–t path, s—t flow, a TSP tour, etc.). The value of an optimal solution to the network optimization problem P in time period t is a function of the flows on the arcs in that period and denoted by $v(x^t)$. The objective is to minimize the total cost over the planning period. Thus, the generic formulation of an incremental network design problem is as follows:

$$\min \sum_{t \in \{1,\dots,T\}} v(x^t) + \sum_{t \in \{1,\dots,T\}, a \in A_p} c_a (y_a^t - y_a^{t-1}) \tag{2.1}$$

s.t.

$$x^t \in F(P) \qquad \forall t \in \{1, \dots, T\} \tag{2.2}$$

$$x_a^t \leqslant C_a y_a^{t-1} \qquad \forall a \in A_p, \quad t \in \{1, \dots, T\} \tag{2.3}$$

$$\sum_{a \in A_p} c_a (y_a^t - y_a^{t-1}) \leqslant B^t \qquad \forall t \in \{1, \dots, T\} \tag{2.4}$$

The objective function (2.1) has two components: (1) the total cumulative value of the solutions to the network optimization problem solved in each period of the planning horizon and (2) the total cumulative cost of the network expansions carried out during the planning horizon. Constraints (2.2) ensure that the solution in each period of the planning horizon has the required structure (i.e., the structure that characterizes solutions to the network optimization problem). Constraints (2.3) ensure that flow on an arc occurs only when the arc has been built in any of the previous periods and that the flow on the arc does not exceed the capacity of the arc. Constraints (2.4) ensure that the cost of building arcs in a period does not exceed the budget available for construction in that period of the planning horizon.

Incremental network design problems have characteristics in common with various network design problems. A brief review of some relevant literature is given below.

3. Literature review

Network design is a fundamental optimization problem and has a rich research tradition. The seminal paper by Magnanti and Wong (1984) discusses many of its features, applications, models, and algorithms, with an emphasis on network design in transportation planning. Kerivin and Mahjoub (2005) survey many network design problems studied in telecommunications. The paper by Magnanti and Wong (1984) mentions "Time Scale" as one of the characteristics of a network design problem that can vary in different planning environments, e.g., transportation and water resource design decisions have long-term effects whereas communication system designs frequently are more readily altered. Not withstanding, the paper focuses exclusively on single-period or single-stage network design problems. Recently, the interest in multi-period or multi-stage network design problems in the area of transportation planning has picked up, partly because it better meets practitioners needs, as in many environments network design decisions span planning periods of up to 25 years and the intermediate network configurations are of concern as well as the final network configuration, see for example Kim, Kim, and Song (2008) and Ukkusuri and Patil (2009). The research reported in Kim et al. (2008) and Ukkusuri and Patil (2009) focuses specifically on traffic networks and the network optimization problem solved in each period is a user equilibrium model. In addition, Ukkusuri and Patil (2009) consider the stochasticity and elasticity of traffic demand. Because the nature and characteristics of user equilibrium models are different from the more traditional network optimization problems that we are interested in, such as the shortest path, maximum flow, and multicommodity flow problem, the resulting bi-level optimization problems are different from the incremental network design problems as well. Furthermore, the focus in Kim et al. (2008) and Ukkusuri and Patil (2009) is primarily on gaining insight into the impact of the timing of network expansions, whereas our focus is on gaining an understanding of the theoretical complexity of incremental network design problems and their approximability.

A class of network design problems where construction over time has been studied extensively is dynamic facility location. The recent review Arabani and Farahani (2012) is completely dedicated to dynamic facility location.

An example of an incremental network design problem that has received a lot of attention recently is the transformation of an electrical power grid into a smart grid (e.g., DeBlasio & Tom, 2008; Farhangi, 2010; Mahmood, Aamir, & Anis, 2008; Momoh, 2009), where it is often the case that resource and budget constraints allow only a limited number of upgrades per time period (e.g., annually). Network design over time also occurs naturally in disruption management, where the functioning of critical infrastructures needs to be restored after a disruption due to environmental, technological or intentional damage to system components (e.g., Lee, Mitchell, & Wallace, 2007, 2009; Matisziw, Murray, & Grubesic, 2010). In this context, Guha, Moss, Naor, and Schieber (1999), Averbakh (2012), and Averbakh and Pereira (2012) focus on the *recovery times* of nodes, which is the first time a node is connected to a special *depot* node. Recovery time is also the objective in the work of Xu et al. (2007) on the restoration of a power network after an earthquake.

Closer to the problem proposed in this paper, in the sense that the focus is on optimizing the cumulative performance of the network over time, is Matisziw et al. (2010) on a multi-objective approach to network restoration where the performance of the network is measured by connectivity, and Cavdaroglu, Hammel, Mitchell, Sharkey, and Wallace (2013) on maximizing the cumulative flow through a set of interdependent networks.

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