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Discrete Optimization

Optimal deterministic algorithms for some variants of Online Quota Traveling Salesman Problem

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ABSTRACT

This paper is concerned with the Online Quota Traveling Salesman Problem. Depending on the symmetry of the metric and the requirement for the salesman to return to the origin, four variants are analyzed. We present optimal deterministic algorithms for each variant defined on a general space, a real line, or a half-line. As a byproduct, an improved lower bound for a variant of Online TSP on a half-line is also obtained.

1. Introduction

The Quota Traveling Salesman Problem (QTSP) is a generalization of the well-known Traveling Salesman Problem (TSP), and can be described as follows. Let M be a metric space with a given origin o and a distance function $d(\cdot,\cdot)$ such that for all $x,\ y,\ z \in M$:

(i) $d(x,y) \geqslant 0$, where "=" holds if and only if $x = y$;
(ii) $d(x,y) = d(y,x)$;
(iii) $d(x,y) \leqslant d(x,z) + d(z,y)$.

Given a set V of vertices in M, a weight w_v associated with each $v \in V$, and a quota Q, we are asked to find a vertex subset $V' \subseteq V$ and a tour on $V' \cup \{o\}$ for a salesman starting from o, such that $\sum_{v\in V'} w_v \geqslant Q$ and the tour is the shortest. The path-version of QTSP (pQTSP for short) is defined similarly as QTSP by replacing the tour with a path that starts from the origin o and goes through the vertex set V'.

In QTSP, if the metric space is replaced by a quasi-metric space, which is equipped with a quasi-metric function satisfying the above conditions (i) and (iii) but not necessarily obeying the condition (ii) of symmetry, we obtain the Asymmetric Quota Traveling Salesman Problem (AQTSP). Clearly, AQTSP is a generalization of QTSP. We use pAQTSP to denote the path-version of AQTSP. Quasi-metric spaces arise naturally in a number of situations, for example, in a city including one-way streets, or on mountain roads, where traveling up hill takes longer than down hill. Particularly, a real line is a quasi-metric space if we define the asymmetrical distance function

$$d(x,y) = \begin{cases} |x-y|, & x \leqslant y, \\ c|x-y|, & x > y, \end{cases} \tag{1}$$

where $c > 0$ is a measure of the asymmetry of the space. If $c = 1$, the real line turns into a metric space.

Another generalization of QTSP is to incorporate release times with vertices, where vertex v can only be visited at or after its release time r_v, the salesman can travel at most at unit speed, and the objective is to find a vertex subset V' reaching the quota Q and a tour on $V' \cup \{o\}$ to minimize the makespan, i.e. the time by which the salesman has visited all the vertices in V' and returned to the origin o. Note that when all vertices are released at time zero, the makespan is just the length of the tour. QTSP with release times is called Offline QTSP if the release times, weights and positions of the vertices are known to the salesman already at time zero. However, the assumption that all the information is completely known a priori is unreasonable in many situations, and there are a lot of applications in which the information is revealed in real-time. These cases can be dealt with appropriately by online models. In Online QTSP, each vertex v presents at its release time r_v, but the release time, weight and even existence of the vertex are known only after its presence. Online/Offline pQTSP, AQTSP, pAQTSP can be defined similarly as Online/Offline QTSP. Note that QTSP, pQTSP, AQTSP, pAQTSP are special cases of the corresponding offline problems in which all vertices are

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released at time zero. In this paper, we mainly investigate the above-mentioned online variants of QTSP.

We only care about the deterministic online algorithms, i.e., the algorithms without coin tosses. The performance of a deterministic online algorithm is usually evaluated by its competitive ratio, i.e., the worst-case ratio of the objective value of the solution produced by the algorithm to the optimal value of the corresponding offline problem. We call γ a (deterministic) lower bound of the online problem if there is no deterministic algorithm with competitive ratio less than γ for it. If the competitive ratio of some deterministic online algorithm matches a lower bound of the online problem, then the algorithm is called the best (or optimal). See Borodin and El-Yaniv (1998) or Fiat and Woeginger (1998) for a thorough study of online algorithms.

Awerbuch, Azar, Blum, and Vempala (1998) proposed an $O(\log^2(\min(Q,n)))$-approximation algorithm for QTSP. According to the results of Ausiello, Leonardi, and Marchetti-Spaccamela (2000), the approximation ratio can be improved to 10. Later, Chaudhuri, Godfrey, Rao, and Talwar (2003) devised, for any $\epsilon > 0$, a $(2+\epsilon)$-approximation algorithm for the Q-path problem in which the objective is to find a shortest path with two prescribed ending vertices going through a set of vertices reaching the quota Q. Note that the Q-path problem generalizes both QTSP and pQTSP. Chekuri, Korula, and Pál (2008) considered some related orienteering problems, and gave a $(2+\epsilon)$-approximation algorithm to find a path with length restriction such that the total weight of vertices visited is maximized. See Vansteenwegen, Souffriau, and Oudheusden (2011) for a survey about the orienteering problem. For Offline QTSP, Ausiello, Bonifaci, and Laura (2008b) gave a $(1+\rho)$-approximation algorithm, where ρ is the best available approximation ratio for QTSP.

Ausiello, Demange, Laura, and Paschos (2004) introduced Online QTSP and devised an algorithm for it with competitive ratio 2ρ provided that a ρ-approximation algorithm is available for QTSP. They also proved a lower bound of 2 when the metric space is the real line, which implies that their algorithm is optimal by using an exact algorithm to solve QTSP. For the real line case, there exists a simple polynomial exact algorithm for QTSP. By contrast, any existing exact algorithm for general QTSP takes exponential time. Moreover, the authors proposed a 3/2-competitive polynomial algorithm for the half-line case as well as a matching lower bound.

Offline TSP is a special case of Offline QTSP in which all vertices have to be visited, but Online TSP is not a special case of Online QTSP since the quota Q in Online QTSP is known a priori and the total number of the vertices in Online TSP is unknown. For Online TSP, both Ausiello, Feuerstein, Leonardi, Stougie, and Talamo (2001) and Lipmann (2003) gave a lower bound of 2, and Ascheuer, Krumke, and Rambau (2000) designed a $\frac{1}{4}(4\rho+1+\sqrt{1+8\rho})$-competitive algorithm by using a ρ-approximation algorithm for TSP, which is optimal by plugging in an exact algorithm for TSP. If the metric space is a real line, Ausiello et al. (2001) provided a lower bound of $\frac{1+\sqrt{17}}{8}\approx 1.64$, while Lipmann (2003) devised an optimal polynomial algorithm. For Online TSP on a half-line, Blom, Krumke, de Paepe, and Stougie (2001) presented a 3/2-competitive polynomial algorithm as well as a matching lower bound.

For the real line and half-line cases of Online pTSP (the path-version of Online TSP, also called Nomadic Online TSP in the literature), Lipmann (2003) proved the lower bounds 2.03 and 1.63, respectively. The author also developed a $\left(\sqrt{2}+\rho\right)$-competitive algorithm for the problem on a general metric space, provided a ρ-approximation algorithm for pTSP. Later, Bonifaci (2007) proposed a 2-competitive algorithm for the half-line case.

For Online Asymmetric TSP, Ausiello, Bonifaci, and Laura (2008a) showed a lower bound of $\frac{3+\sqrt{5}}{2}\approx 2.618$ and proposed a $\frac{1}{2}(2\rho+1+\sqrt{1+4\rho})$-competitive algorithm, where ρ is the approximation ratio for ATSP. Their algorithm is optimal by solving ATSP to optimality. The authors also proved that the corresponding path-version has no online algorithm with competitive ratio bounded by a constant.

In this paper, we focus on Online QTSP and AQTSP as well as their path-versions. First we present polynomial $(1+\rho)$-competitive algorithms for them, provided that ρ-approximation algorithms are available for the corresponding offline problems with zero release times. Our algorithms improve on the 2ρ-competitive algorithm for Online QTSP given by Ausiello et al. (2004). Then we provide algorithms for Online AQTSP and pAQTSP on a half-line with competitive ratios $\frac{2c+1}{c+1}$ and $\min\{1+c,2\}$, respectively, where c is the measure of asymmetry of the space. We also show that all the above algorithms are optimal by devising lower bounds matching the competitive ratios. Combining those existing results with ours, we obtain a complete mapping of the best deterministic lower bounds and algorithms for the variants concerned of Online QTSP. All the results are summarized in Table 1. In addition, as a byproduct, we improve the lower bound of Online pTSP on a half-line from 1.63 to 2, which implies the 2-competitive algorithm in Bonifaci (2007) is optimal.

The remainder of the paper is organized as follows. In Section 2, we describe some notations. In Section 3, we present algorithms for Online AQTSP, pAQTSP, QTSP and pQTSP and analyze their competitive ratios. In Section 4, we prove the lower bounds of Online AQTSP, pQTSP, pAQTSP on a real line or a half-line and Online pTSP on a half-line.

2. Notations

Here we introduce some notations used throughout the paper. Let σ be an instance of Online QTSP, pQTSP, AQTSP or pAQTSP. We use $T(\sigma)$ to denote the solution given by some algorithm for σ as well as the corresponding makespan, $p(t)$ to denote the posi-

Table 1
The best deterministic lower bounds and algorithms for variants of Online QTSP.

	General space		Real line		Half-line	
	LB	Algorithm	LB	Algorithm	LB	Algorithm
Online QTSP	2	$1+\rho$	2	2	3/2	3/2
Online AQTSP	2	$1+\rho$	2	2	$\frac{2c+1}{c+1}$	$\frac{2c+1}{c+1}$
		(Theorem 1)	(Theorem 11)	(Theorem 3)	(Theorem 7)	(Theorem 4)
Online pQTSP	2	$1+\rho$	2	2	2	2
Online pAQTSP	2	$1+\rho$	2	2	$\min\{1+c,2\}$	$\min\{1+c,2\}$
		(Theorem 2)	(Theorem 12)	(Theorem 3)	(Theorem 8)	(Theorem 6)

Notes: The results for real line and half-line hold for each value $c > 0$ of the measure of asymmetry.
Unreferenced results can be obtained from the others.
The results for Online QTSP on real line and half-line have been given in Ausiello et al. (2004).