European Journal of Operational Research 238 (2014) 741–759

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

An integrated approach for earthwork allocation, sequencing and routing

R.L. Burdett, E. Kozan *

Decision Sciences Discipline, Queensland University of Technology, Australia

ARTICLE INFO

Article history:
Received 27 August 2013
Accepted 27 April 2014
Available online 9 May 2014

Keywords:
Earthworks
Road design
Terrain modelling
Meta-heuristics

ABSTRACT

Planning techniques for large scale earthworks have been considered in this article. To improve these activities a "block theoretic" approach was developed that provides an integrated solution consisting of an allocation of cuts to fills and a sequence of cuts and fills over time. It considers the constantly changing terrain by computing haulage routes dynamically. Consequently more realistic haulage costs are used in the decision making process. A digraph is utilised to describe the terrain surface which has been partitioned into uniform grids. It reflects the true state of the terrain, and is altered after each cut and fill. A shortest path algorithm is successively applied to calculate the cost of each haul, and these costs are summed over the entire sequence, to provide a total cost of haulage. To solve this integrated optimisation problem a variety of solution techniques were applied, including constructive algorithms, meta-heuristics and parallel programming. The extensive numerical investigations have successfully shown the applicability of our approach to real sized earthwork problems.

1. Introduction

1.1. Background

Numerous plans are needed to organise construction operations. These plans must include all facets of the construction process and can be for daily, weekly, or monthly time periods. A strategic plan of forthcoming earthworks is particularly necessary in linear infrastructure projects (such as road and rail construction), and its creation is focussed upon in this article. For background information concerning road construction and optimisation, we direct readers to Jha, Schonfeld, and Jong (2006).

A strategic plan of earthworks is a description of what earth goes where, and is otherwise characterised as an assignment of cuts to fills. Each assignment of this nature is known as a cut–fill pairing, and signifies a volume of material that must be excavated from a specified location (or area) and hauled to another. Without loss of generality there are $N!$ cut–fill assignments to choose from, if N is the number of cut and fills, and no other technical constraints have been imposed. A sequence of these pairings can be used to describe when each location is cut or filled over time. There are similarly $N!$ orderings of the cut–fill pairs.

In past research, cuts and fills occurred within predefined "sections". The following articles are noteworthy for taking this approach: Hare, Koch, and Lucet (2011), Henderson, Vaughan, Jacobson, Wakefield, and Sewell (2003), Ji, Borrmann, and Wimmer (2011, 2010), Moselhi and Alshibani (2009), Shah and Dawood (2011), Son, Mattila, and Myers (2005). Though conceptually easy to grasp and use, these sections can be quite vague in terms of size, shape and location and can also be quite "coarse". For a variety of reasons both practical and technical, the consideration of what occurs within sections has not often been included in the process of constructing an earthwork allocation plan (EAP). For example, once cutting and filling activities have started, the state of the terrain (i.e. the current elevations), within sections, are not often modelled. It should however be noted that in practice, contractors may perform very short moves with an excavator. These can be interpreted as free hauls to some extent as they do not involve trucks, and the cost is negligible compared to other "standard" hauls.

1.2. Research objective

The objective of this article is to find high quality integrated earthwork solutions, consisting of an allocation of cuts to fills, a sequence of cuts and fills, and a path (route) for each haul. This problem is technically challenging as it is highly combinatorial and large problem instances occur in practice. For example this

---

* Corresponding author. Tel.: +61 731381029; fax: +61 731382310.
E-mail addresses: r.burdett@qut.edu.au (R.L. Burdett), e.kozan@qut.edu.au (E. Kozan).

http://dx.doi.org/10.1016/j.ejor.2014.04.036
0377-2217/© 2014 Elsevier B.V. All rights reserved.

integrated decision problem has a search space of at least $(N!)^{N!}$. It should also be noted that there are many possible paths to choose from for each cut–fill pair. Hence the search space is vastly bigger again. In this article, in our largest test problem, $N \cong 1500$. This problem is NP-hard and the proof of this is given in Appendix A.

In this article a different approach to earthworks planning has been considered. The notion of sections has been discarded, and discrete 3D blocks are utilised. Each block is a container of earth. It has a specific location in 3D and has a common volume. After each cut or fill, the elevation within each block can be updated. Stacks of blocks also introduce inherent cutting and filling precedence, and these can be incorporated for improved realism. The concept of blocks has been motivated heavily by recent research outcomes in the field of mining and earthmoving. A recent study by Cheng and Jiang (2013) has shown the advantage of 3D modelling and computation of earthwork volumes, as opposed to simpler 2D approaches. The only justification for section based approaches is when survey precision is poor and GPS data is not provided.

The costs associated with earthworks are dynamic. This is because the terrain dictates fuel consumption, emissions and other cost metrics. Since it is constantly being altered by cutting and filling activities, the distances, gradients and surfaces between different points are constantly varying. In response to this feature, a *dynamic objective function* is proposed (in Section 2) for evaluating earthmoving decisions (i.e. hauls). This objective function provides the shortest path for each haul according to the current state of the terrain. The shortest paths are required in order to (i) optimise material haulage and (ii) to evaluate the real merit of moving material between any two cut and fill block locations. The shortest path approach determines the optimal path between two block locations on the surface according to a given metric like cost, distance, work, and time.

For modelling the terrain, and for evaluating alternative earthwork solutions, a graph model is presented in Section 2. That approach attempts to accurately evaluate the real cost of haulage. Issues of solution feasibility are considered as is routing between alternative "disconnected" project sites. An optimisation methodology and several solution approaches are then proposed in Section 3. These approaches are then applied to ten case studies of varying size, in Section 4, to show their applicability and to identify which settings and options are best.

1.3. Related work

Blocks have been used heavily in mining applications in recent years. Examples include: Bienstock and Zuckerberg (2010), Boland, Dumitrescu, Froyland, and Gleixner (2007), Caccetta and Hill (2003), Cullenbine, Wood, and Newman (2011), Lamghari and Dimitrakopoulos (2012), L'Heureux, Gamache, and Soumis (2013), Ramazan (2007), and Underwood and Tolwinski (1998).

This article is also related to our recent work on earthworks planning. However a very different approach is taken here that is more comprehensive. For example this article provides a full 3D model of earthworks and fully models the changing terrain that occurs during construction. In Burdett and Kozan (2014) mathematical (LP and MIP) models were developed to identify a more detailed assignment of cuts to fills. These models require the project site to be partitioned into blocks. This approach was shown to be superior to previous section based alternatives, as the position of earth at different elevations is more realistically and accurately modelled. These block models are generic and are suitable for both 2D and 3D scenarios. The physics concept "work" was also used as a proxy for fuel consumption. Two earthwork decision problems were introduced. The first considers the movement of whole blocks of material, whereas the second partial amounts. In this article we consider the first problem because from a practical perspective, it is more realistic to dig up a discrete block of earth, and to shift it to one specified place, instead of trying to accurately break it up into many smaller parts, and to transport them to many specific locations. In practice different soil types are also mixed up and cannot be easily divided on site. In Burdett and Kozan (2013a), 3D road construction problems were considered. A cross section replication approach, which solves a 2D variant of the earthwork allocation problem, and replicates the solution across the y-axis, was found to be a viable and effective approach in specific situations. An approach that partitions the domain into separate cross sections was also investigated.

Several other researchers have also considered integrated earthwork problems. Henderson et al. (2003) treated the earthworks planning problem as a symmetric travelling salesman problem. In particular the shortest route cut and fill problem (SRCFP) was defined. It seeks to find a route (i.e. an alternating path of cuts and fills, beginning and ending at the same cut location) for a single earthmoving vehicle that minimises the total distance travelled between cut and fill locations. It is a variant of this problem that we consider in the present article. Lim, Rodrigues, and Zhang (2005) also considered the SRCFP problem and developed a superior meta-heuristic approach that incorporates features from Simulated Annealing (SA) and Tabu Search (TS). They also added some important innovations such as multiple vehicles and a makespan objective.

Hare et al. (2011) considered earthworks planning to minimise construction costs. They extended MILP models for planning by incorporating time steps and the concept of "blocks", which are obstacles or other topographical feature that need to be removed before earth can be moved across or over them. These blocks are different to the containers of earth that are used in this article. In that article costs are static and are independent of the time step. That article is noteworthy as it introduces an earthworks planning approach that, at least conceptually, and albeit in a lesser way, handles the changing terrain that occurs during the construction process.

The article by Nassar and Hosny (2012) considered an earthwork planning problem that is more similar to the one addressed in this article. That article is of particular note because they consider conceptually the changing state of the terrain and the sequence of cuts and fills. They define their problem as the least-cost route cut and fill sequencing problem (LCRCFP). It was formulated as a mixed binary optimisation problem and solved using a traditional branch-and-bound method and a particle swarm optimization (PSO) technique. Their model however demonstrated that a different decision making problem is solved to the one in this article, though the description is similar. They view work in a standard way, as the distance multiplied by the volume of earth moved, i.e. the tonne kilometres. In this article we view work as the physics concept, i.e. force multiplied by distance. Work is then multiplied by the number of trips made to transport a specific volume of earth. The advantages of our approach have been described in Burdett and Kozan (2014). Nassar and Hosny have however suggested that vehicle performance characteristics like, grade resistance, rolling resistance, and fuel consumption rates should be incorporated, and on the basis of time and effort available in practice. This is done in this article. Nassar and Hosny described terrain locations using stations and sections and areas by grids (cells). They appear to use static distances, and not actual Euclidean distances over the changing terrain, as they stated. They do not include changes in the elevation at each location or in each cell. Nassar and Hosny solved many test problems (1D and 2D) to demonstrate their approach. The problems considered however were relatively small, and most were randomly generated. In this article we solve real 3D problems of a larger nature.