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Discrete Optimization

New results for the Directed Profitable Rural Postman Problem

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ABSTRACT

We study a generalization of the Directed Rural Postman Problem where not all arcs requiring a service have to be visited provided that a penalty cost is paid if a service arc is not crossed. The problem, known as Directed Profitable Rural Postman Problem, looks for a tour visiting the selected set of service arcs while minimizing both traveling and penalty costs. We add different valid inequalities to a known mathematical formulation of the problem and develop a branch-and-cut algorithm that introduces connectivity constraints both in a "lazy" and in a standard way. We also propose a matheuristic followed by an improvement heuristic (final refinement). The matheuristic exploits information provided by a problem relaxation to select promising service arcs used to solve optimally Directed Rural Postman problems. The ex-post refinement tries to improve the solution provided by the matheuristic using a branch-and-cut algorithm. The method gets a quick convergence through the introduction of connectivity cuts that are not guaranteed to be valid inequalities, and thus may exclude integer feasible solutions.

All proposed methods have been tested on benchmark instances found in literature and compared to state of the art algorithms. Results show that heuristic methods are extremely effective outperforming existing algorithms. Moreover, our exact method is able to close, in less than one hour, all the 22 benchmark instances that have not been solved to optimality yet.

1. Introduction

Let $G(V,A)$ be a directed graph, where $V = \{0,\ldots,n\}$ is the node set (node 0 represents the depot), and $A = \{(i,j) \mid i \neq j, i,j \in V\}$ is the arc set. Let $R \subseteq A$ be a subset of arcs requiring a service (*service arcs*). A penalty p_{ij} is assigned to each arc $(i,j) \in R$ and a traveling cost c_{ij} is associated with each arc $(i,j) \in A$. The penalty represents the cost that has to be paid if the corresponding service arc is not selected. The problem looks for a tour starting and ending at the depot so that total traveling and penalty costs are minimized. Since the objective of minimizing traveling and penalty costs can be reformulated as maximizing the difference between the profit gained by serving required arcs and the traveling costs, the problem is called Directed Profitable Rural Postman Problem (DPRPP). Moreover, the problem is NP-hard generalizing the well-known Directed Rural Postman Problem (DRPP) where all service arcs have to be visited in the tour (see Eiselt, Gendreau, & Laporte (1995) and Dror (2000)).

The problem belongs to the class of arc routing problems with profits. Whereas vast literature exists for node routing problems with profits (see Feillet, Dejax, & Gendreau (1981) and Vansteenwegen, Souffriau, & Van Oudheusden (2011)), there are still very few contributions which can be found on their arc routing counterpart (see Corberán & Prins (2010) for a very recent annotated bibliography). As far as we know, Malandraki and Daskin (1993) are the first authors to consider arc profits by introducing a directed version of the prize-collecting arc routing problem. In their problem a prize can be collected from each arc multiple times, and the size of the prize decreases with the number of collections. Feillet, Dejax, and Gendreau (2005) introduce the Profitable Arc Tour Problem (PATP). The objective function of the PATP maximizes the profit located on arcs minus the travel costs using a fleet of unlimited vehicles with no capacity constraint, but an upper bound on the length of each tour. No depot is considered and the profit is available on arcs for a limited number of times. More recently, Aráoz, Fernández, and Zoltan (2006) introduce the Privatized Rural Postman Problem (PRPP). The PRPP looks for a tour starting and ending at the depot while maximizing the difference between the profits gained by serving required edges and the crossing costs. The PRPP can be seen as the undirected version of the DPRPP. The authors study problem polyhedral properties identifying some dominance relations. Later on, Aráoz, Fernández, and Meza (2009) rename the problem Prize-collecting Rural Postman Problem. The authors identify upper bounds by solving iteratively

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relaxed models with a small number of inequalities and adding violated cuts through exact separation procedures. They also provide lower bounds for the problem generating feasible solutions using the 3T heuristic by Fernández, Meza, Garfinkel, and Ortega (2003). Their problem formulation uses two sets of binary variables indicating when edges are served and when only crossed, respectively. They show that the number of times an edge can be crossed in an optimal solution is never higher than 2, thus generalizing a known result for the Rural Postman Problem (RPP) (see Christofides, Campos, Coberán, & Mota (1981)).

Since the work by Aráoz et al. (2006), many other authors have studied different variants of the prize-collecting RPP. Examples are the clustered prize-collecting ARP by Aráoz, Fernández, and Franquesa (2009), and the windy clustered prize-collecting ARP by Corberán, Fernández, Franquesa, and Sanchis (2011). More recently, Black, Eglese, and Wøhlk (2013) study the time-dependent prize-collecting Arc Routing Problem (TD-PARP) inspired to the PRPP but with the addition of real constraints. The graph is directed and traversing costs depend on the time they are crossed. Moreover, the problem allows for more requests between the same two nodes. Finally, in the literature there are problems where profits are placed on nodes and arcs. This is the case of the one-period Bus Touring Problem (BTP) introduced by Deitch and Ladany (2000). In the BTP the profits are non-negative attractiveness values and the objective is to maximize the total attractiveness of the tour by selecting a subset of nodes (sites) to be visited and arcs (scenic routes) to be traveled subject to constraints on touring time, cost and total distance.

The problem studied in this paper originates in the domain of the transportation service procurement where companies need to decide which customers to serve directly and which to assign to external operators paying an outsourcing cost. In Guastaroba, Mansini, and Speranza (2009) the problem, introduced under the name of Shipper's Lane Selection Problem (SLSP), considers a shipper that has to decide which lanes (service arcs) to pick out for a direct service with its own vehicle, and which to assign conveniently to external carriers paying an outsourcing cost (the penalty). Since lanes outsourcing is made through an auction, the problem objective is the minimization of the sum of traveling costs and outsourcing costs plus the fixed cost of the auction set-up. The authors provide a mathematical formulation using binary instead of integer variables, thus allowing a single crossing for each arc. Other works can be found in this application area almost all referring to node instead of arc routing problems. See for instance Chu (2005) who first analyzes the problem of deciding how many private vehicles to employ in truckload mode (and then defining their routing) and how many external carriers to use in less than truckload mode when the customer demand is more than the available vehicle capacity. A slightly different problem formulation is also analyzed by Côté and Potvin (2009) who propose a Tabu Search algorithm to solve it. More recently, the SLSP has been reformulated by Archetti, Guastaroba, and Speranza (2012) who call it Directed Profitable Rural Postman Problem and introduce an effective Tabu Search algorithm with the addition of an ex-post ILP-refinement. They also provide a set of benchmark instances for the problem many of which have not been solved to optimality up to now.

This paper provides different contributions. We introduce a tighter problem formulation than the one proposed in Archetti et al. (2012) adding different valid inequalities and use it to develop a branch-and-cut algorithm. We implement the proposed exact method separating the connectivity constraints both using the standard separation based on the solution of max-flow problems at each node of the search tree and excluding subtours only when integer solutions are found (lazy separation). The combination of the new problem formulation with the lazy separation of the connectivity constraints results to be the best performing approach allowing to close all the 22 instances not solved to optimality by Archetti et al. (2012) in less than one hour of computing time. To guarantee a fair comparison with the literature we test our algorithms on a comparable PC than the one exploited in Archetti et al. (2012), and use the same version of MIP solver (CPLEX 10.1) the authors used in the paper. Note that in this CPLEX version no subroutine for lazy constraints is available.

The mathematical formulation of the problem has also been used to construct an effective matheuristic and a final improvement method. Matheuristics are hybrid methods that combine heuristics and mathematical programming approaches providing a very promising alternative to meta-heuristics in solving hard combinatorial problems (we refer interested readers to Maniezzo, Stützle, & Voss (2009)). Our matheuristic exploits information provided by a problem relaxation to identify subsets of "promising" service arcs used to solve to optimality Directed Rural Postman problems. Each DRPP is formulated on a *restricted graph* induced by the selected service arcs (Auxiliary Problem). The matheuristic consists of two main routines, one considering subsets of service arcs of increasing size and the other one of decreasing size. Finally, the improvement heuristic (final refinement) receives as input the best integer solution provided by the matheuristic and tries to improve it exploiting the branch-and-cut framework of a common MILP solver by inserting connectivity cuts that may not be valid inequalities and thus may eliminate feasible solutions. When an integer solution is found, the method looks for larger subtours and eliminates them forcing at least one arc to leave each of them, whereas this may not be the case in an optimal solution. On the contrary, small subtours are excluded through the insertion of standard violated inequalities as in an exact method. A defined parameter establishes when a subtour is considered large. Choosing this parameter, the method looks for the best trade-off between a quick convergence obtained through a massive introduction of connectivity cuts not guaranteed to be valid inequalities (large subtours) and a more substantial selection of violated cuts as in an exact method. In the first case, a higher efficiency is paid in terms of a lower performance, while in the second one better solution values can be obtained at the cost of a larger computing time.

The paper is organized as follows. In Section 2 the mathematical formulation of the problem, different valid inequalities and some simple properties are discussed. We also describe the mathematical formulation of the Auxiliary Problem used to solve optimally the DRPP when the subset of service arcs to be visited is given. Section 3 is devoted to solution algorithms description, whereas in Section 4 we test both our exact algorithm and the heuristic approaches on benchmark instances introduced in Archetti et al. (2012) and compare their performance with state of the art algorithms. In Section 5 some conclusions and future developments are drawn.

2. Mathematical formulation

We start by describing the integer linear programming formulation for the DPRPP. The model uses $O(|A|)$ integer variables x to indicate the number of times each arc is crossed and $O(|A|)$ binary variables y to select arcs to serve. Finally, $O(|V|)$ binary variables z are used to formulate connectivity constraints:

$$x_{ij} \geqslant 0 \quad \text{number of times arc } (i,j) \text{ is crossed} \quad (i,j) \in A,$$

$$y_{ij} := \begin{cases} 1 & \text{if arc } (i,j) \text{ is served} \\ 0 & \text{otherwise;} \end{cases} \quad (i,j) \in R,$$

$$z_j := \begin{cases} 1 & \text{if node } j \text{ is visited} \\ 0 & \text{otherwise.} \end{cases} \quad j \in V \setminus \{0\}.$$

The problem can be formulated as follows: