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Discrete Optimization

A perturbation based variable neighborhood search heuristic for solving the Vehicle Routing Problem with Simultaneous Pickup and Delivery with Time Limit

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ABSTRACT

The Vehicle Routing Problem with Simultaneous Pickup and Delivery with Time Limit (VRPSPDTL) is a variant of the basic Vehicle Routing Problem where the vehicles serve delivery as well as pick up operations of the clients under time limit restrictions. The VRPSPDTL determines a set of vehicle routes originating and terminating at a central depot such that the total travel distance is minimized. For this problem, we propose a mixed-integer mathematical optimization model and a perturbation based neighborhood search algorithm combined with the classic savings heuristic, variable neighborhood search and a perturbation mechanism. The numerical results show that the proposed method produces superior solutions for a number of well-known benchmark problems compared to those reported in the literature and reasonably good solutions for the remaining test problems.

1. Introduction

The Vehicle Routing Problem (VRP) is a combinatorial optimization problem seeking to service a set of clients from a central depot with a homogeneous fleet of capacitated vehicles. This problem aims to determine a set of vehicle routes originating and terminating at the central depot and visiting all clients such that the total travel distance is minimized. A variant of the VRP is the Vehicle Routing Problem with Backhauls (VRPB) where the vehicles dispatch goods (*delivery*) from the depot to linehaul clients and collect goods (*pickup*) from backhaul clients to the depot. The VRPB can be basically categorized into four classes (Parragh, Doerner, & Hartl, 2008):

- VRP with Clustered Backhauls (VRPCB): the vehicles first dispatch delivery goods to linehaul clients, then collect pickup goods from backhaul clients.
- VRP Mixed linehauls and Backhauls (VRPMB): the vehicles serve delivery or pick up operations to linehaul or backhaul clients in any sequence.
- VRP with Divisible Delivery and Pickup (VRPDDP): clients demanding both delivery and pickup operations could be visited separately.
- VRP with Simultaneous Pickup and Delivery (VRPSPD): clients demanding both delivery and pickup operations have to be visited once.

For extended variants of Pickup and Delivery Problems (PDP) see Berbeglia, Cordeau, Gribkovskaia, and Laporte (2007) and Parragh et al. (2008) and for Location Routing Problems (LRP) see Prodhon and Prins (2014). In the literature, the VRPSPD was first proposed by Min (1989). In the VRPSPD, the current load of the vehicle has to be checked at each client to ensure that the vehicle capacity is not violated. This problem can be classified according to the notation proposed by Berbeglia et al. (2007). They use the 3-tuple notation [*Structure*|*Visits*|*Vehicles*] to describe the characteristics of a PDP. In this notation, *structure* represents the number of origins and destinations of goods, *visits* represents information on the way pickup and delivery operations are performed at clients, and *vehicles* represents the number of vehicles employed. The VRPSPD investigated in this paper can be characterized as [*1-M-1*|*PD*|*m*] where *1-M-1* indicates one-to-many-to-one problems, i.e. goods are initially available at the depot and are transported to clients, and goods available at the clients are transported to the depot; *PD* means each client being visited exactly once for a combined pickup and a delivery operation; *m* denotes the availability of more than one vehicle (multi-vehicle problem).

The VRPSPD can also be categorized into three classes as follows:

- VRPSPD with Maximum Distance Length (MLVRPSPD): a maximum voyage distance constraint for returning to the central depot is imposed for each of the vehicles.

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- VRPSPD with Time Windows (VRPSPDTW): the vehicles have to start their service at the clients' locations between a given earliest and latest time.
- VRPSPD with Time Limit (VRPSPDTL): the vehicles have to return to the central depot before a time deadline is reached.

In this study, we consider the VRPSPDTL that additionally includes service times for the clients and a maximum total duration (travel + service time) restriction for the vehicles as an extension of the VRPSPD.

Dethloff (2001) defined the VRPSPD as an NP-hard combinatorial optimization problem, meaning that practical large-scale problem instances are hard to solve by use of exact solution methodologies within acceptable computational times. In the VRPSPDTL the objective and constraints are the same as in the VRPSPD, except for the service time limit of vehicles for returning to the central depot. This makes the problem more complicated due to the difficulty in controlling the voyage duration of the vehicles in addition to the service time of the clients along the route. As a result, this problem can be described as NP-hard, as well. In the literature, the interest was therefore on the development of heuristic or meta-heuristic solution approaches.

Since 1989, many heuristic and meta-heuristic solution approaches for solving VRPSPD benchmark problems have been proposed. See Goksal, Karaoglan, and Altiparmak (2013), Subramanian, Drummond, Bentes, Ochi, and Farias (2010), Zachariadis, Tarantilis, and Kiranoudis (2009) and Zachariadis and Kiranoudis (2011) for recent studies on the VRPSPD, Montané and Galvão (2006) and Zhang, Tian, Zhang, and Liu (2008) for the MLVRPSPD and Liu, Xie, Augusto, and Rodriguez (2013) and Mingyong and Erbao (2010) for the VRPSPDTW. Furthermore, some authors consider the VRPSPD as a special case of the VRPDDP where vehicles are allowed to visit clients separately for delivery and pick-ups (Nagy, Wassan, Speranza, & Archetti, 2014). Although, Salhi and Nagy (1999) defined benchmark problem instances for the VRPSPD with "time limit", since then most of the studies disregarded this case. A possible reason could be the increased complexity of the problem. So far just a few studies considered benchmark problems under time limit restrictions. Some authors proposed heuristic and meta-heuristic implementations for the VRPSPD as well as VRPSPDTL. These studies are as follows: Cluster Insertion Heuristics (CIH) by Salhi and Nagy (1999), Insertion Based Heuristics (IBH) by Dethloff (2001), Alternating Heuristic Algorithms (ALT) by Nagy and Salhi (2005), Large Neighborhood Search (LNS) by Ropke and Pisinger (2006), Tabu Search (TS) by Montané and Galvão (2006), Reactive Tabu Search (RTS) by Wassan , Wassan, and Nagy (2008), Ant Colony System (ACS) by Gajpal and Abad (2009), Particle Swarm Optimization (PSO) by Ai and Kachitvichyanukul (2009), Savings Based Ant Algorithm (SBAA) by Catay (2010) and Nearest Sweep with Perturbation (NSP) by Jun and Kim (2012).

Subramanian and Cabral (2008) presented the first investigation that deals with the pure VRPSPDTL considering the CMT 6-7-8-9-10-13-14 X&Y benchmark problems of Salhi and Nagy (1999). The authors proposed an Iterated Local Search (ILS) procedure in order to solve this problem.

In this study, we propose a perturbation based neighborhood search (PVNS) approach for solving the VRPSPDTL. The remainder of this study is organized as follows. In Section 2, we present a model formulation for the problem at hand. Next, the solution procedure is developed in Section 3. Detailed numerical results are presented in Section 4. Finally, conclusions are drawn and suggestions for further research are given in Section 5.

2. The Vehicle Routing Problem with Simultaneous Pickup and Delivery with Time Limit

The problem considered in this study is designing the network of service vehicles, e.g. simultaneously dispatching/collecting cargo parcels from a central post station to/from regional post stations via trucks, simultaneously dispatching/collecting containers from a hub port to/from feeder ports via containerships, simultaneously dispatching/collecting passengers from a continental center airport to/from national airports via airplanes, etc. In this context, the VRPSPDTL can be stated as follows: A set of clients is located on a distribution network where clients require both delivery and pickup operations. Each client has to be served once for both operations with a given fleet of identical capacitated vehicles. Each vehicle leaves the central depot carrying the total amount of goods that it has to deliver and returns to the depot carrying the total amount of goods that it must pick-up. Each client also has a specified service time which is the loading and unloading operation time of the vehicle at the client. Therefore, the voyage time of a vehicle is the sum of total travel time of the route and total service time of the clients. In order to determine the vehicle schedules and the staffing balance, each vehicle has to finish its voyage before the maximum allowed duration is reached.

A mixed-integer linear programming (MILP) formulation for the VRPSPDTL is proposed with the following notation by extending the VRPSPD formulation of Montané and Galvão (2006):

Indices

$i, j \in N$	the set of nodes (clients and depot (0))
$k \in K$	the set of vehicles

Parameters

R	maximum allowed voyage duration of vehicles
Q	maximum loading capacity of a vehicle
v	average travel speed of a vehicle
n	number of nodes, i.e., $n = \|N\|$
c_{ij}	distance between nodes i and j
s_i	service time at client i
d_i	delivery goods demand of client i
p_i	pick-up goods demand of client i

Decision variables

x_{ij}^k	1, if the arc (i,j) belongs to the route served by vehicle k; 0, otherwise
y_{ij}	pick-up goods transported on arc (i,j)
z_{ij}	delivery goods transported on arc (i,j)
π_i	sub-cycle elimination variable used for client i

The model formulation is given as follows.

$$\min \sum_{k\in K}\sum_{i\in N}\sum_{i\in N} c_{ij}x_{ij}^k \tag{1}$$

s.t.

$$\sum_{i\in N}\sum_{k\in K} x_{ij}^k = 1 \quad \forall j \in N/\{0\} \tag{2}$$

$$\sum_{i\in N} x_{ij}^k - \sum_{i\in N} x_{ji}^k = 0 \quad \forall j \in N, k \in K \tag{3}$$

$$\sum_{j\in N/\{0\}} x_{0j}^k \le 1 \quad \forall k \in K \tag{4}$$

$$\sum_{i\in N/\{0\}} x_{i0}^k \le 1 \quad \forall k \in K \tag{5}$$

$$\sum_{i\in N} y_{ji} - \sum_{i\in N} y_{ij} = p_j \quad \forall j \in N/\{0\} \tag{6}$$

$$\sum_{i\in N} z_{ij} - \sum_{i\in N} z_{ji} = d_j \quad \forall j \in N/\{0\} \tag{7}$$

$$y_{ij} + z_{ij} \le Q \sum_{k\in K} x_{ij}^k \quad \forall i, j \in N \tag{8}$$

$$\sum_{i\in N}\sum_{j\in N} \frac{c_{ij}}{v} x_{ij}^k + \sum_{i\in N/\{0\}}\sum_{j\in N} s_i x_{ij}^k \le R \quad \forall k \in K \tag{9}$$