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Emergency lateral transshipments in a two-location inventory system with positive transshipment leadtimes

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ABSTRACT

We consider a single-echelon continuous review inventory system for spare parts with two parallel locations. Each location faces independent Poisson demand and backorders are allowed. In this paper we consider the possibility of lateral transshipments between the locations. The transshipment leadtime is positive and deterministic, and there is an additional cost for making a transshipment. We suggest a transshipment policy which is based on the timing of all outstanding orders, and develop and solve a heuristic model by using theory and concepts from doubly stochastic Poisson processes and also partial differential equations. A simulation study indicates that our heuristic works very well, and that the relative cost increase of disregarding the transshipment leadtime may be quite high. Our results also indicate that it is, in general, worth the effort of reducing the transshipment leadtime, even if it is already relatively short.

1. Introduction

Consider a manufacturing company that provides its customers with spare parts. For example, the packaging-machine producing company Tetra Pak Technical Service acts also as a provider of spare parts to its customer's when a random failure has occurred in a packaging machine situated at a customer site. When a failure at a machine has occurred it is crucial to replace the failured spare part quickly in order to maximize the up-time of the customers production system. Evidently, long waiting times for spare parts cause long production stand stills, which in turn lead to large costs in terms of loss of production and revenue. Another type of costs that are common in these kind of production systems are inventory perishing costs. Consider for example packaging of dairy products. Then, due to the nature of the product, if the production down-time exceeds a certain time limit, a whole batch of the dairy product is wasted. Consequently, the backorder cost in such situations may be very high.

A possible option to reduce downtimes at the customer sites is to introduce lateral transshipments between the customer sites. Given the development of information systems it is possible and relatively cheap to keep track of units in supply chains. For instance, by using so called RFID (Radio Frequency Identification) it is possible to update residual leadtimes continuously. Very few earlier papers dealing with lateral transshipments use information about the timing of outstanding orders. Moreover, most earlier papers in this area assume that transshipment leadtimes are negligible. The most relevant papers in connection with our model are Axsäter (2003a) and Yang, Dekker, Gabor, and Axsäter (2013). Axsäter (2003a) considers a continuous time inventory system with negligible transshipment leadtimes where lateral transshipments are allowed. Assuming full information about the state of the system (such as residual replenishment leadtimes) an optimal transshipment rule is derived given that no further transshipments will take place. As a heuristic, this rule is repeatedly used. Hence, in Axsäter (2003a) a decision to make a transshipment is based on the timing of outstanding orders and the status of inventory levels. Yang et al. (2013) are dealing with a spare parts inventory system with positive transshipment leadtimes. Using a heuristic they introduce so called customer oriented service levels, which means that a customer order is assumed to be satisfied if it is completed within a certain time window. Other papers that assume non-negligible transshipment leadtimes are, e.g., Wong, Van Houtum, Cattrysse, and Van Oudheusden (2006) and Reijnen, Tan, and van Houtum (2011).

This paper extends the literature in several directions. First, we provide a new way of modeling lateral transshipments in inventory systems with constant replenishment leadtimes based on information about the age of the units in the system. We also relax the assumption of negligible transshipment leadtimes by assuming a constant positive transshipment leadtime. Although our model is quite similar to the model presented in Yang et al. (2013), there are some important differences. First, our solution procedure is more general. In fact, it turns out that the solution method in Yang et al. (2013) is a

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special case of our approach. Yang et al. (2013) use the results obtained from a model without lateral transshipments and then replace the demand rates with adjusted demand rates when incorporating lateral transshipments in the model. However, in our model we derive the exact solution when allowing lateral transshipments conditioned on the adjusted demand rates. In this paper we consider backorder costs (or sometimes called down-time costs) instead of a pre-specified target service level. To the best of our knowledge, it is relatively common in practice that service providing companies in situations like this one, are obliged to pay penalty fees, per unit of time, whenever they cannot deliver service as agreed upon. Therefore, we consider time dependent down-time costs instead of customer oriented service levels. Another reason for doing this is the practical problem of measuring service levels (e.g., fill rates) effects over a short period of time. To measure fill rates correctly, historical data for a relatively long interval of time are needed, which may be a problem at many companies when evaluating the performance of the system after a relatively short period of time.

This paper can also be viewed as an extension and generalization of the model considered in Moinzadeh (1989). Moinzadeh (1989) considers an $(S-1, S)$ inventory system with Poisson demand where it is assumed that an arriving customer is backordered with a given probability α and therefore lost with probability $1-\alpha$. Under these assumptions Moinzadeh (1989) develops an exact solution for the expected total cost. In this paper, we will as a spin-off from our lateral transshipment model develop an exact solution to a slightly more general version than considered in Moinzadeh (1989).

The literature concerning lateral transshipments in inventory systems is relatively rich. For a very recent and extensive review see, e.g., Paterson, Kiesmüller, Teunter, and Glazebrook (2011). In general, inventory models including lateral transshipments are considerably more difficult to analyze than their counterparts with regular supply. This is the case since the state (e.g., inventory on hand, residual leadtimes of items, etc.) at the receiving location depends on the state at the sending location, and vice versa. In fact, it is interesting to note that no (non-trivial) exact solution exists for models considering continuous review centralized inventory systems with lateral transshipments and *deterministic* replenishment leadtimes. However, when the replenishment leadtimes are assumed to follow an exponential distribution (which is quite artificial), there exist several models dealing with lateral transshipments that provide exact solutions. This simple fact demonstrates the extreme complexity of the problem when assuming the more realistic case of non-exponential leadtimes.

As mentioned, there is a large family of papers concerning continuous review centralized inventory systems with lateral transshipments, and a majority of these are based on exponentially distributed replenishment leadtimes and negligible transshipment leadtimes. Axsäter (1990) falls under this category and uses a queueing theoretical approach in order to derive expressions for fill rates, etc. Some other papers in the same spirit as Axsäter (1990) are, e.g., Alfredsson and Verrijdt (1999), Grahovac and Chakravarty (2001), Kukreja, Schmidt, and Miller (2001) and Olsson (2009, 2010). In a recent paper, Zhang and Archibald (2011) develop a semi-Markov decision model with lateral transshipments and phase-type distributed leadtimes. Since a phase-type distributed random variable approaches a deterministic value when the number of phases grows larger, this is an interesting extension to the common assumption of exponentially distributed leadtimes. Another recent paper, Van Wijk, Adan, and van Houtum (2009), derive the optimal transshipment policy assuming exponential replenishment leadtimes.

A related family of inventory systems concerns so called emergency replenishments instead of lateral transshipments. In these systems it is possible to replenish from an outside supplier, which provides a fast emergency replenishment but incur additional costs. In fact, it turns out that the model developed for lateral transshipments in this paper can also be used for the purpose of analyzing inventory systems with emergency replenishments. A few papers related to ours, considering emergency replenishments that take pipeline information into account are, e.g., Song and Zipkin (2009) and Howard, Reijnen, Marklund, and Tan (2014).

In the next section, we formulate our model in detail and give a list of relevant notation. In Section 3 we present our solution technique, and in Section 4 we use this technique in order to derive expressions for various performance measures. In Section 5 we evaluate our model in a numerical study and give some implications from both a tactical and strategical point of view. Finally, in Section 6, a concluding discussion is provided.

2. Model formulation

We consider a single-echelon continuous review inventory system with two parallel locations (location 1 and location 2). Each location faces independent Poisson demand and backorders are allowed. In this paper we will concentrate on spare part products, which are often expensive low demand products. This means that it is reasonable to use $(S-1, S)$ replenishment policies. Hence, in a standard situation without lateral transshipments, location i orders directly from an outside supplier when a demand occurs at location i, and the order will arrive after a fixed leadtime. Let us introduce some useful notation:

λ_i – Customer arrival intensity at location i
L_i – Regular replenishment leadtime for location i
ℓ – Transshipment leadtime ($\ell < L_i$)
S_i – Order-up-to level at location i
IL_i – Inventory level at location i
h_i – Holding cost per unit and unit time at location i
b_i – Backorder cost per unit and unit time at location i
τ – Transshipment cost per unit
$\triangleq$ – Equals per definition
x^+ – $\max(0, x)$
x^- – $\max(0, -x)$
$\text{Po}(m)$ – Poisson distribution with mean m.

In this paper we consider the possibility of lateral transshipments between two parallel locations. Since we want to utilize information about the pipeline situation, let us keep track of the ages of items in the system. For the sake of convenience and clarity we separate the system into two sub-systems. Sub-system 1 consists of the items in stock at location 1 and the items which have not yet arrived at location 1 from the supplier. Sub-system 2 is defined in the same way as sub-system 1, but for location 2 instead. Let $T_1^{(i)}, T_2^{(i)}, \ldots, T_{S_i}^{(i)}$, $i \in \{1, 2\}$, represent the age of the items in sub-system i which are not assigned for waiting customers, i.e., the S_i youngest items in sub-system i (note that the other items in the sub-system are already assigned for waiting customers). We define $T_1^{(i)}$ as the age of the oldest item and $T_{S_i}^{(i)}$ as the age of the youngest item. Hence, we have the order $0 \le T_{S_i}^{(i)} < T_{S_i-1}^{(i)} < \cdots < T_1^{(i)} < \infty$. In this model we assume that the age of an item is measured from the time an order for an item is placed.

As mentioned, most previous papers concerning lateral transshipments make the simplifying assumption that transshipment leadtimes are negligible. In this paper we will, however, relax this assumption and assume that transshipment leadtimes are positive and fixed. Obviously, additional problems arise regarding how to construct a reasonable transshipment rule. In earlier papers where the transshipment leadtime is disregarded it is commonly assumed that a transshipment is always realized as soon as an arriving customer faces zero stock on hand, while another location has positive stock on hand. This is the classical complete pooling strategy. However, in the case with a positive transshipment leadtime it may be unwise to