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Measuring the bullwhip effect for supply chains with seasonal demand components

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ABSTRACT

A bullwhip measure for a two-stage supply chain with an order-up-to inventory policy is derived for a general, stationary SARMA$(p, q) \times (P, Q)_s$ demand process. Explicit expressions for several SARMA models are obtained to illustrate the key relationship between lead-time and seasonal lag. It is found that the bullwhip effect can be reduced considerably by shortening the lead-time in relation to the seasonal lag value.

1. Introduction

When retailers submit orders to their suppliers in response to demand, they must carefully balance inventory levels and expected future demand. Ensuring that the inventory can meet consumer demand often results in higher fluctuations in the ordering process than the variability of demand would initially suggest. This phenomenon is called the bullwhip effect and can depend on many factors such as the ordering policy, the lead-time for an order to arrive, and even the demand process itself. Lee, Padmanabhan, and Whang (1997a, 1997b) have illustrated the existence of this effect at firms such as Proctor & Gamble Co. and Hewlett-Packard Company. The distortion of information as orders go upstream in a supply chain has been studied from a mathematical perspective by examining the magnitude of the bullwhip effect under various settings. In this paper, we consider a single-item, two-stage supply chain with an order-up-to inventory policy and derive a bullwhip effect measure for a general class of demand models: seasonal autoregressive and moving average processes (i.e., SARMA$(p,q)\times(P,Q)_S$). We use this theory to show the crucial links among the magnitude of the bullwhip effect, the lead-time, and seasonal lag.

There is a large body of work showing that demand has seasonal components. Wray (1958, pp. 4) writes, for example, that toy purchases are seasonal with demand rising around the Christmas holidays. Baltas (2005) and Heien (1983) examine the seasonal behavior of the demand for nondurable goods such as laundry detergent, food, and energy. Board (2008) and Heien (1983) also examine housing and Board (2008) continues his analysis for other durable goods. Finally, for a more general discussion examining cyclical demand across industries and countries, see Beaulieu, MacKie-Mason, and Miron (1992).

Furthermore, seasonal demand has been previously considered within the operations and supply chains context – just not, as far as we know, in the bullwhip literature. For example Crowston, Hausman, and Kampe (1973) and McClain and Thomas (1977) focus on production planning for goods which are in demand on a seasonal basis. Sethi and Cheng (1997) and Aviv and Federgruen (2001) study inventory systems for products with seasonal demand as well. Finally, Bradley and Arntzen (1999) combine production planning and inventory policy along with capacity in the presence of seasonal demand. Given this large body of literature and that the demand process is a key driver of the bullwhip effect, we feel it is critical to incorporate demand seasonality in our measurements.

This paper is organized as follows. We begin with a review of the existing bullwhip effect literature in Section 2. In Section 3, we outline the supply chain framework for one supplier and one retailer where the latter employs an order-up-to inventory policy and mathematically define the bullwhip measure. In Section 4, we describe the SARMA$(p,q)\times(P,Q)_S$ model. Sections 3 and 4 provide the foundation for the main result. Next, in Section 5, we derive the bullwhip measure for the general SARMA demand case. We apply our theorem to the AR(1), MA(q), and ARMA(1,1) settings to connect our new results to previous work. In Section 6, we consider the SARMA$(1,0)_S$,

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$SARMA(0, 1) \times (1, 0)_s$, and $SARMA(1, 0) \times (0, 1)_s$ cases, deriving formulas analytically from the main theorem. We examine the conditions under which the bullwhip effect exist focusing specifically on the key relationship between the seasonal lag and the lead-time between placing and receiving an order. Moreover, we show how to reduce the bullwhip effect by exploiting this relationship. Finally, we provide some concluding remarks in Section 7.

2. Background

For a general overview of the bullwhip effect and its causes, see Lee et al. (1997a). Luong (2007), Luong and Phien (2007), and Duc, Luong, and Kim (2008) derive explicit expressions for the bullwhip measure for the AR(1), AR(p) and ARMA(1,1) demand processes respectively. We will be generalizing their results in this paper. Graves (1999) has examined the ARIMA(0,1,1) case as well. Gilbert (2005) and Galmaan (2006) examine the bullwhip effect on orders and inventory for ARMA and ARIMA models. Gilbert (2005) considers a slightly different setup than what we do here. Ma and Wu (2014) look at AR(1) and simple ARMA models for multi-stage supply chains. However, the effects of seasonality are not considered.

A second approach focuses on comparing the magnitude of the bullwhip effect among different forecasting methods. For example, Chen, Ryan, and Simchi-Levi (2000) forecast demand with exponential smoothing. Zhang (2004b) compares three forecasting methods: minimizing the mean-squared forecasting error (used by many researchers including us), moving average, and exponential smoothing. He finds that the bullwhip measure can vary based on the forecasting method applied. Hayya, Kim, Disney, Harrison, and Chatfield (2006) examine forecasting methods with an additional focus on the impact of lead-times in the bullwhip analysis. Marchena (2011) examines estimation methods for ARMA demand models.

A final stream of research concentrates on ways to reduce the bullwhip effect. Alwan, Liu, and Yao (2003) study demand forecasting methods which can reduce the bullwhip effect specifically for AR(1) and ARMA(1,1) demand processes. Wang, Jia, and Takahashi (2005) investigate how unforeseen events, such as demand shocks, influence their extended bullwhip effect measure. They also discuss how information sharing between the retailer and supplier can help eliminate the bullwhip effect altogether. Gill and Abend (1997) show this in action through their case study on Wal-Mart Stores, Inc. Chatfield, Kim, Harrison, and Hayya (2004) investigate the bullwhip effect especially in the case of variable lead-times for an order-up-to, two-stage supply chain process using simulation. They find that information sharing can reduce the bullwhip effect. Using agent-based modeling, Zarandi, Pourakbar, and Turksen (2008), derive ordering polices which can decrease the bullwhip effect. Dejonckheere, Disney, Lambrecht, and Towill (2003) and Disney and Towill (2003) examine how to reduce the bullwhip effect using various fractional ordering policies. Gaalman (2006) extends this to the ARMA demand case when the lead-time is one period.

We note that in none of these cases does seasonal demand play a part in the analyses, which (a) has been shown to exist (Baltas, 2005; Beaulieu et al., 1992; Board, 2008; Heien, 1983; Wray, 1958) and (b) is incorporated in other supply chain research (Aviv & Federgruen, 2001; Bradley & Arntzen, 1999; Crowston et al., 1973; McClain & Thomas, 1977; Sethi & Cheng, 1997). We demonstrate in this paper that the relationship between lead-time and seasonal lags are crucial to bullwhip effect reduction.

3. Supply chain framework

Consider a single-item, two-stage supply chain with one supplier and one retailer. Assume that the retailer employs an order-up-to inventory policy. In this section, we borrow notation from Luong (2007) as we describe the supply chain framework. If we index the fixed-length inventory review period by the subscript t, an order placed at time t arrives in $t + L$ periods; therefore, L is the fixed lead-time. At the start of period t, the retailer places an order of quantity Q_t to the supplier based both on previously realized demand and future forecasted demand. The order placed at time $t - L$ arrives and D_t units are sold (i.e., D_t is demand at time t). Note that L includes the inventory review period and is assumed to be a positive integer ($L \geq 1$).

Q_t depends on forecasts of the lead-time demand, denoted as D_t^L, from time t to time $t + L - 1$, the period before the order placed at time t arrives. Now, let $\hat{D}_{t+s}$ be the minimum mean squared error predictor of D_{t+s} based on $D_{t-1}, D_{t-2}, \ldots$. Then, we define lead-time demand and its forecast as:

$$D_t^L = D_t + D_{t+1} + \cdots + D_{t+L-1} \tag{1}$$

$$\hat{D}_t^{L,t-1} = \hat{D}_t^{t-1} + \hat{D}_{t+1}^{t-1} + \cdots + \hat{D}_{t+L-1}^{t-1}. \tag{2}$$

The term $\hat{D}_t^{t-1}$ denotes the forecasted demand at time t with realized demand known up to time $t - 1$. Likewise, $\hat{D}_{t+1}^{t-1}$ is the forecasted demand for time $t + 1$ also with realized demand known up to time $t - 1$, and so forth.

The order-up-to level S_t is computed using the forecasted demand for the L periods before the order arrives plus the safety (buffer) stock:

$$S_t = \hat{D}_t^{L,t-1} + \text{safety stock}. \tag{3}$$

Next, the order quantity is calculated as follows:

$$Q_t = S_t - (S_{t-1} - D_{t-1}) = S_t - S_{t-1} + D_{t-1} \tag{4}$$

$$= \sum_{i=0}^{L-2} \underbrace{\hat{D}_{t+i}^{t-1} - \hat{D}_{t+i}^{t-2}}_{(A)} - \underbrace{\left(\hat{D}_{t-1}^{t-2} - D_{t-1}\right)}_{(B)} + \underbrace{\hat{D}_{t+L-1}^{t-1}}_{(C)} \tag{5}$$

$$= \hat{D}_t^{L,t-1} - \hat{D}_{t-1}^{L,t-2} + D_{t-1}. \tag{6}$$

This calculation is best interpreted with (5), although we use (6) in subsequent computations. Component (A) denotes the change in the forecast for time $t + i$ between periods $t - 2$ and $t - 1$. For example, if (A) is positive, more demand is expected in period $t + i$ than originally forecasted. Component (B) is the difference between the realized demand and the final forecast for time $t - 1$. Predicted demand for the final period before the order is placed in time t is component (C). Note that the safety stock is irrelevant in the bullwhip calculations as it is canceled out in (4).

Finally, the bullwhip effect is the propagation of the variation in demand up the supply chain. Mathematically, the bullwhip measure function $B(\cdot)$ is defined as:

$$B(\cdot) = Var[Q_t]/Var[D_t]. \tag{7}$$

If $B(\cdot) > 1$, then the bullwhip effect is present.

4. SARMA(p, q)×(P, Q)$_s$ model

As demand is seasonal for a variety of products (see Baltas, 2005 or Wray, 1958), using SARMA models may be more appropriate to describe it than ARMA models alone. This addition allows us to consider a much larger class of demand processes, resulting in more accurate models for demand, and therefore, more accurate bullwhip measurements. A downside is that the equations become complex and additional data is required to fit such models. We describe the model form next.

Let D_t be a stationary, SARMA(p, q)×(P,Q)$_s$, written using the Box and Jenkins (1970) notation:

$$\Phi_P\left(B^s\right)\phi(B)D_t = \Theta_Q\left(B^s\right)\theta(B)a_t \tag{8}$$

where p denotes the order of the autoregressive (AR) component, q is the order of the moving average (MA) component, P is the order of the seasonal autoregressive element, Q is the order of the seasonal