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Which is better for replacement policies with continuous or discrete scheduled times?

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ABSTRACT

In order to maintain a unit that is running successive works with cycle times, this paper classifies its replacement policies into three types: (Type I) Replacement with interrupted cycles; (Type II) replacement with complete works; and (Type III) replacement with incomplete works. Type I is typically done at a continuous time T while Type II is executed at a discrete number N of working cycles. Type III is proposed as an improvement of Type I, which can be done at discrete working cycles. For each type, age and periodic replacement models are respectively observed. It is shown that Type I is more flexible than Type II and costs less than Type III. However, modified replacement costs, i.e., without penalty of operational interruptions, are obtained for Types II and III as critical points at which their policies should be adopted. All discussions are presented analytically and numerical examples are given when each cycle time is exponential and the failure time has a Weibull distribution.

1. Introduction

Manufacturing systems with performance degradation and maintenance strategy are commonly encountered in practice. To seek balance between failure losses and maintenance costs, preventive maintenances have been investigated extensively in the past decades. Recently, a methodical survey of maintenance models and their optimizations in reliability theory was done (Nakagawa, 2005). Other published books (Frenkel, Karagrigoriou, Lisnianski, & Kleyner, 2013; Kobbacy & Murthy, 2008; Kuo & Zuo, 2003; Manzini, Regattieri, Pham, & Ferrari, 2010; Nakagawa, 2008; Osaki, 2002; Pham, 2011; Wang & Pham, 2007) collected many kinds of reliability models and maintenance policies in theory and their applications in industrial systems.

Although some categories of maintenance policies (Wang, 2002; Wu & Zuo, 2010) and several overviews of recent maintenance models (Ahmad & Kamaruddin, 2012; Shafiee & Chukova, 2013; Wang, 2012) have been surveyed, it can be classified in most above models that systems are maintained or replaced before failure at some continuous measures (i.e., age, operating time, running distance, damage threshold, condition level, repair cost, etc.), at some discrete quantities (i.e., number of usage, works, shocks, faults, failures, repairs, etc.), or at some bivariate maintenance times such as optimized t^* along with n^* (Chen, 2012; Sheu & Chang, 2010; Sheu, Li, & Chang, 2012; Zhang, Yam, & Zuo, 2007). Taking aircraft engines as an example, appropriate maintenances are usually scheduled at a total hours of operations or at a specified number of flights since the last major overhaul (Duchesne & Lawless, 2000). This also could be applied to maintenance strategies for the systems whose operational functions deteriorate with either or both causes of age and use (Nakagawa, 2008). In other words, it is advisable to maintain or replace such systems before catastrophic failure when their operations reach a certain usage time or a specified number of uses.

However, there is no theoretical study to tell us whether or not maintenances scheduled at continuous measures are more predominant than those at discrete quantities, which is the first problem to be considered in this paper. On the other hand, the system usually executes works with successive cycles, e.g., number N of random cycles, so that it is difficult to specify the quality of maintenance or replacement policies (Wu & Clements-Croome, 2005) without considering the factor of executed works. The replacement policies that are scheduled along with working cycles (Chen, Mizutani, & Nakagawa, 2010) were modeled. Replacement models planned with continuous or discrete policies using the approach of whichever triggering event occurs last were discussed recently (Zhao, Nakagawa, & Zuo, 2014). However, when the system is operating for some successive

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works without stops, it is better to do maintenance after several works are completed even though the maintenance time has arrived (Zhao & Nakagawa, 2013). That is, maintenances scheduled at continuous times need to be modified according to discrete cycles to satisfy work completions, which is the second problem to be considered in this paper.

In order to maintain a unit that is running successive works with cycle times, this paper classifies the following replacement policies scheduled at continuous and discrete times into three types:

I. Replacement with interrupted cycles: The unit is replaced before failure at a planned time T, which is called continuous age replacement (CAR).
II. Replacement with complete works: The unit is replaced before failure at completion of the Nth working cycle, which is called discrete age replacement (DAR).
III. Replacement with incomplete works: The unit is replaced before failure at the first completion of some working cycle over a planned time T, which is called age replacement with overtime (ARO).

For each type, periodic replacement policies are also considered in a similar way, that is, continuous periodic replacement (CPR), discrete periodic replacement (DPR), and periodic replacement with overtime (PRO) are defined and modeled in the following sections. Joint determinations of Types I and II, and Types II and III for age and periodic replacement models under the assumption "whichever occurs first" are also discussed.

Obviously, policy defined in Type III is an improvement of that in Type I, which can be done at discrete working cycles. Policy in Type II guarantees completion of all works, e.g., N works, while part of works can be completed under Type III, but cycles are always interrupted under Type I. It will be shown that Type I is more flexible than Type II and costs less than Type III, when replacement costs for three types are originally supposed as order of "Type I ≤ Type II" and "Type I ≤ Type III". However, the order of penalty cost for operational interruption would be "Type I > Type III > Type II". So that how to find modified replacement costs (i.e., the replacement cost without penalty of operational interruption) for Types II and III as critical points at which their policies should be adopted become the main purpose of this paper.

2. Age replacement

Consider a unit that is running successive works with cycle times Y_j $(j = 1, 2, \ldots)$, where random variables Y_j have an identical distribution $G(t) \equiv \Pr\{Y_j \le t\}$ with finite mean $1/\theta$ $(0 < \theta < \infty)$. The unit deteriorates with operation and has a failure time X according to a general distribution $F(t) \equiv \Pr\{X \le t\}$ with finite mean μ $(0 < \mu < \infty)$, which is independent of Y_j.

To model the following replacement policies, we denote $G^{(j)}(t)$ $(j = 1, 2, \ldots)$ be the j-fold Stieltjes convolution of $G(t)$ with itself and $G^{(0)}(t) \equiv 1$ for $t \ge 0$, $g(t) \equiv \mathrm{d}G(t)/\mathrm{d}t$, $g^{(j)}(t) \equiv \mathrm{d}G^{(j)}(t)/\mathrm{d}t$, and $r_j(t) \equiv g^{(j)}(t)/[1 - G^{(j)}(t)]$, where $r_j(t)\mathrm{d}t$ represents the probability that the unit completes the jth working cycle in $[t, t + \mathrm{d}t]$, given that it has been running for the jth cycle at time t. In addition, let $h(t) \equiv f(t)/\overline{F}(t)$ be the failure rate of $F(t)$ and $H(t) \equiv \int_0^t h(u)\mathrm{d}u$ be cumulative hazard rate, where $f(t) \equiv \mathrm{d}F(t)/\mathrm{d}t$ and $\overline{F}(t) \equiv 1 - F(t)$. It is also assumed that $h(t)$ increases strictly from $h(0) = 0$ to $h(\infty) \equiv \lim_{t\to\infty} h(t)$.

Suppose that when the unit fails, its failure is immediately detected, and then CR (corrective replacement) is done. As PR (preventive replacement) policies, we firstly model the joint policies of Types I and II into age replacement, i.e., the unit is replaced before failure at a planned time T $(0 < T \le \infty)$ or at the completion of Nth $(N = 1, 2, \ldots)$ working cycle, whichever occurs first. The expected replacement cost rate is, from Chen et al. (2010),

$$C_1(T; N) = \frac{c_T + (c_F - c_T)\int_0^T [1 - G^{(N)}(t)]\mathrm{d}F(t) + (c_N - c_T)\int_0^T \overline{F}(t)\mathrm{d}G^{(N)}(t)}{\int_0^T [1 - G^{(N)}(t)]\overline{F}(t)\mathrm{d}t}, \tag{1}$$

where c_F is CR cost at failure, and c_T $(c_F > c_T)$ and c_N $(c_F > c_N)$ are PR costs at time T and at number N, respectively.

Clearly, $C_1(T; N)$ includes the respective Type I and Type II age replacement models as $T \to \infty$ or $N \to \infty$. That is, if $N = \infty$, then the policy corresponds to PR that is made at time T, which is called age replacement (Barlow & Proschan, 1965) or CAR in Type I, and the expected cost rate is

$$C_1^{\mathrm{CAR}}(T) \equiv \lim_{N\to\infty} C_1(T; N) = \frac{c_F - (c_F - c_T)\overline{F}(T)}{\int_0^T \overline{F}(t)\mathrm{d}t}. \tag{2}$$

On the other hand, if $T = \infty$, the policy corresponds to PR that is made at number N of working cycles, which is called DAR in Type II, and the expected cost rate is

$$C_1^{\mathrm{DAR}}(N) \equiv \lim_{T\to\infty} C_1(T; N) = \frac{c_F - (c_F - c_N)\int_0^\infty G^{(N)}(t)\mathrm{d}F(t)}{\int_0^\infty [1 - G^{(N)}(t)]\overline{F}(t)\mathrm{d}t} \quad (N = 1, 2, \ldots). \tag{3}$$

2.1. Optimal policies

We find optimal time T_1^* and number N_1^* which minimizes $C_1^{\mathrm{CAR}}(T)$ in (2) and $C_1^{\mathrm{DAR}}(N)$ in (3), respectively. From the results of (Nakagawa, 2005), if $h(\infty) > c_F/[\mu(c_F - c_T)]$, then there exists a finite and unique T_1^* $(0 < T_1^* < \infty)$ which satisfies

$$\int_0^{T_1^*} [h(T) - h(t)]\overline{F}(t)\mathrm{d}t = \frac{c_T}{c_F - c_T}, \tag{4}$$

and the resulting cost rate is

$$C_1^{\mathrm{CAR}}(T_1^*) = (c_F - c_T)h(T_1^*). \tag{5}$$

From the inequality $C_1(N + 1) - C_1(N) \ge 0$,

$$Q_N \int_0^\infty [1 - G^{(N)}(t)]\overline{F}(t)\mathrm{d}t - \int_0^\infty [1 - G^{(N)}(t)]\mathrm{d}F(t) \ge \frac{c_N}{c_F - c_N} \quad (N = 1, 2, \ldots), \tag{6}$$

where

$$Q_N(T) \equiv \frac{\int_0^T [G^{(N)}(t) - G^{(N+1)}(t)]\mathrm{d}F(t)}{\int_0^T [G^{(N)}(t) - G^{(N+1)}(t)]\overline{F}(t)\mathrm{d}t} \le h(T),$$

and $Q_N \equiv \lim_{T\to\infty} Q_N(T)$. It has been proved (Chen et al., 2010) that if Q_N increases strictly with N to Q_∞, then the left-hand side of (6) also increases to $\mu Q_\infty - 1$. Thus, if $Q_\infty > c_F/[\mu(c_F - c_N)]$, then there exists a finite and unique minimum N_1^* $(1 \le N_1^* < \infty)$ which satisfies (6), and the resulting cost rate is

$$(c_F - c_T)Q_{N_1^*-1} < C_1^{\mathrm{DAR}}(N_1^*) \le (c_F - c_T)Q_{N_1^*}. \tag{7}$$

In particular, when $G(t) = 1 - \mathrm{e}^{-\theta t}$, i.e., $G^{(N)}(t) = \sum_{j=N}^\infty [(\theta t)^j/j!]\mathrm{e}^{-\theta t}$ $(N = 1, 2, \ldots)$,

$$Q_N(T) = \frac{\int_0^T (\theta t)^N \mathrm{e}^{-\theta t}\mathrm{d}F(t)}{\int_0^T (\theta t)^N \mathrm{e}^{-\theta t}\overline{F}(t)\mathrm{d}t}$$

increases strictly with N and converges to $h(T)$ as $N \to \infty$, and hence Q_N increases with N to $h(\infty)$. Thus, if $h(\infty) > c_F/[\mu(c_F - c_N)]$, then there exists a finite and unique minimum N_1^* $(1 \le N_1^* < \infty)$ which satisfies (6).

Note that if the condition $h(\infty) > c_F/[\mu(c_F - c_i)]$ $(i = T, N)$ holds, there exist both finite T_1^* and N_1^* that minimize their respective cost rates.