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Decision Support

Calibration of agricultural risk programming models

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ABSTRACT

Positive Mathematical Programming (PMP) is one of the most commonly used methods for calibrating activity programming models. In this article we consider PMP as a calibration method for risk programming models with a mean-variance (E-V) specification. We argue that the restrictive theoretical assumptions employed by typical linear E-V models limit their applicability in analyzing the effects of decoupled payments on agricultural production decisions. Furthermore, the requirement for eliciting a risk aversion coefficient renders such models incompatible with the PMP method. For this reason we propose a nonlinear E-V specification and develop a PMP-based procedure for its calibration which does not aim at introducing (further) nonlinearities in the objective function, but at recovering the "true" distribution of wealth that will allow the final model to reproduce base year observations. We also examine how our approach relates to the recent PMP developments on calibration against elasticity priors and we show how such priors can be used for the calibration of the nonlinear E-V model.

1. Introduction

The effect of uncertainty and risk aversion on decision making has already received much attention in the economic literature and theoretical contributions have examined the behavior of the optimizing firm outside the neoclassical paradigm of parameter certainty and risk-neutrality (e.g. Sandmo, 1971). In the context of mathematical programming, risk has typically been modeled in a linear mean-variance (E-V) framework with an exogenously specified risk aversion parameter under the assumption of expected utility (E-U) maximization.

Although linear E-V farm models have been used extensively in the discipline of agricultural economics, analysts employing them face two serious problems. The first is the availability of data that will allow for a complete characterization of the variance of allocative choices; typically such exercises require empirical distributions on prices and/or outputs which are rarely available at the farm level. For this reason it is common to use distributions from regional or even national data, under the assumption that preferences are homogeneous among farmers, although it is acknowledged that this practice often leads to a biased representation of risk (Gómez-Limón, Arriaza, & Riesgo, 2003).

The second problem is that an efficient calibration method for linear E-V models still does not exist. On the contrary, calibration methods for deterministic programming models have evolved significantly because agricultural economists applying operational research methods in agriculture realized that not all objective function specifications can simulate the observed market outcome (Kutcher & Norton, 1982). This finally led to a distinction between normative and positive models which has come to dominate the literature, especially after the seminal work of Howitt (1995a) on Positive Mathematical Programming (PMP).

PMP assumes that the observed activity mix is optimal and considers calibration problems to be the result of unobserved implicit information that affects economic behavior. This implicit information is integrated into the model by introducing or re-estimating nonlinear terms in the objective function, so that the final nonlinear model exactly reproduces the observed activity vector. Interestingly, the PMP methodology seems to provide an answer (albeit unorthodox) to the previously described problem of data availability, since its fundamental principle is to calibrate programming models by estimating primal or dual specifications of production technology with only minimal data. Accordingly, our first argument in this article is that PMP can be used in an E-V context to estimate the wealth distribution which leads to the observed production choices.[1] Furthermore, we argue that the

[1] This does not imply, however, that we support the estimation of behavioral functions for economic agents from a single observation. Our argument concerns those cases for which behavioral homogeneity, expressed through the selection of a common regional or national wealth distribution, is not a plausible assumption, since it may lead to erroneous model results. We discuss such issues in Section 3 and present an appropriate example in Section 4.

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linear E-V model is not compatible with the standard PMP method and so the analyst should re-examine the suitability of the linear E-V specification. This argument also stems from the theoretical limitations of linear E-V models and more precisely from their restrictive assumption that an agent's response to risk does not depend on the initial level of wealth. As a result of this assumption, such models cannot capture the effect of decoupled support mechanisms on farm production decisions because economic behavior is considered to be invariant to nonrandom profits.

In order to bypass the previous theoretical limitation we propose the use of a nonlinear E-V model with a decreasing absolute risk aversion coefficient (DARA) and we develop a novel PMP method for its calibration. The working hypothesis is that the model may fail to calibrate because of (i) a wrong wealth distribution and (ii) the existence of implicit marginal costs that affect economic behavior. Additionally, we examine how to control the second order properties of the final model by introducing supply elasticity priors into the calibration process, as proposed by recent contributions in the PMP literature (e.g. Mérel, Simon, & Yi, 2011; Mérel & Bucaram, 2010). Our approach differs from existing applications in that we consider a full elasticity matrix, consisting of both own- and cross-supply elasticities. We also show that because of the strong nonlinearities in the DARA model it is necessary to make use of the implicit function theorem, which, however, results in a computationally difficult estimation problem.

The article is structured as follows: Section 2 explains the principles of the PMP method and presents the seminal example of calibrating linear programming (LP) models using nonlinear cost functions. Then we review the techniques for incorporating risk into economic programming models and investigate the methods proposed for their calibration. In Section 3 we examine the reasons for which the nonlinear E-V model may fail to calibrate and provide a detailed presentation of both the proposed PMP method, and of using supply elasticities in the calibration process. A simple farm-level application within the current Common Agricultural Policy (CAP) setting is given in Section 4, in order to demonstrate the model's ability to calibrate and its response sensitivity to changes in the level of decoupled payments received by the farm. The paper ends with a discussion section where we identify the limitations of the nonlinear E-V specification and examine how our approach relates to the critique concerning the PMP method.

2. Materials and methods

2.1. The PMP methodology

The PMP algorithm is a two-step process that begins with the introduction of additional calibration constraints to the initial LP model that force it to replicate base year observations, $\mathbf{x}_0$:

$$\max_{\mathbf{x}\geq 0} \quad \Pi = \mathbf{r}^\top \mathbf{x} - \mathbf{c}^\top \mathbf{x}$$

$$\text{s.t.} \quad \mathbf{A}\mathbf{x} \leq \mathbf{b} \qquad [\mathbf{z}] \tag{1}$$

$$\mathbf{x} \leq \mathbf{x}_0 + \varepsilon \qquad [\mathbf{h}] \tag{2}$$

where Π denotes farm's gross margin to be maximized, $\mathbf{x}$ is the $I \times 1$ vector of unknown activity levels (e.g., hectares), $\mathbf{r}$ is the $I \times 1$ vector of activity revenues and $\mathbf{c}$ the $I \times 1$ vector of average variable costs. Inequality (1) represents the resource constraints, $\mathbf{A}$ is the $M \times I$ matrix of technical coefficients and $\mathbf{b}$ is the $M \times 1$ vector of available resources. The dual values associated with the resource constraints are given by the $M \times 1$ vector $\mathbf{z}$. The calibration constraints (2) bind each activity to its observed level, while $\mathbf{h}$ is the corresponding $I \times 1$ dual vector and ε is a small perturbation term that is used to prevent model degeneration.[2] The model calibrates and its first order conditions with respect to $\mathbf{x}$ are written as:

$$\mathbf{r} - \mathbf{c} - \mathbf{h} - \mathbf{A}^\top \mathbf{z} = 0 \tag{3}$$

The dual vector $\mathbf{h}$ is the key parameter in the PMP method and it is argued that it embodies any type of marginal implicit information on a farmer's production choices, or it may implicitly account for model misspecifications and data errors that can cause calibration problems. Paris and Howitt (1998) interpret $\mathbf{h}$ as a "differential" marginal cost vector that, together with the observed "accounting" variable average cost $\mathbf{c}$, can reveal the "true" variable marginal cost at $\mathbf{x}_0$. As is shown by Howitt (1995a), the necessary and sufficient condition for a model to calibrate is that the objective function be nonlinear in at least some of the decision variables (activities). The PMP algorithm thus aims at transforming the linear objective function into a nonlinear one that integrates all information contained in $\mathbf{h}$, so that the model calibrates without any additional constraints. The nonlinearity is usually sought in the cost term and is introduced in the model by replacing the linear cost function $\mathbf{c}^\top\mathbf{x}$ with a quadratic one, $C(\mathbf{x}) = \mathbf{d}^\top\mathbf{x} + 0.5\mathbf{x}^\top\mathbf{L}\mathbf{x}$, where $\mathbf{d}$ is an $I \times 1$ vector of linear terms and $\mathbf{L}$ is an $I \times I$ positive, semi-definite matrix that is either diagonal or fully specified.[3] The final nonlinear model is written as:

$$\max_{\mathbf{x}\geq 0} \quad \Pi = \mathbf{r}^\top\mathbf{x} - \mathbf{d}^\top\mathbf{x} - 0.5\mathbf{x}^\top\mathbf{L}\mathbf{x} \qquad \text{s.t.} \qquad \mathbf{A}\mathbf{x} \leq \mathbf{b} \quad [\mathbf{z}]$$

and the unknown parameters $\mathbf{d}$ and $\mathbf{L}$ must be estimated so that the model's first order conditions with respect to $\mathbf{x}$ are exactly satisfied at $\mathbf{x}_0$:

$$\mathbf{r} - \mathbf{d} - \mathbf{L}\mathbf{x}_0 - \mathbf{A}^\top\mathbf{z} = 0 \tag{4}$$

Since both the initial bounded model and the final quadratic one calibrate, we can combine Eqs. (3) and (4) and obtain the following equation:[4]

$$\mathbf{d} + \mathbf{L}\mathbf{x}_0 = \mathbf{c} + \mathbf{h} \tag{5}$$

In the second step of the PMP algorithm, Eq. (5) is used for the estimation of the unknown parameters $\mathbf{d}$ and $\mathbf{L}$. This constitutes an ill-posed problem with I equations and $2I$ or $I + I(I+1)/2$ unknown parameters, depending on the form of the $\mathbf{L}$ matrix; if $\mathbf{L}$ is diagonal, several ad hoc methods have been proposed, summarized by Heckelei (2002), while Paris and Howitt (1998) were the first to estimate a fully specified matrix. Their approach was based on the maximum entropy (ME) criterion and is now considered the standard method for estimating a full PMP $\mathbf{L}$ matrix.

2.2. Risk in economic models

The E-V criterion and E-U theory are the principal methods for modeling choices under risk in economics. E-V analysis is based on Markowitz's (1952) pioneering work on portfolio theory and postulates that an economic agent selects an activity mix which minimizes the variance of income, σ^2 (or standard deviation, σ), for a given expected income, μ. On the other hand, E-U theory represents a more axiomatic approach to decision making under risk. It assumes that individuals assign discrete or continuous probability distributions to risky prospects and respond to this risk by maximizing the utility expectation $E[U(\cdot)]$ of the wealth W that these prospects generate. More formally, let W_0 be the non-stochastic part of wealth[5] and α a random

[2] A degeneration problem can occur when the resource constraints $\mathbf{A}\mathbf{x} \leq \mathbf{b}$ contain a land constraint expressed as $\sum_i x \leq \sum_i x_0$, where x and x_0 denote the ith element of $\mathbf{x}$ and $\mathbf{x}_0$ (with $i \in I$). If the perturbation term is omitted, the two constraint sets are linearly dependent.

[3] The quadratic cost function is convex in $\mathbf{x}$, thus leading to decreasing marginal profits. Alternatively, nonlinearities can be introduced in the form of a concave production function (e.g. Howitt, 1995a,b).

[4] This step assumes that the resource shadow price vector $\mathbf{z}$ is the same in both models. The implications of this assumption are discussed in Section 5.2.

[5] In economic literature, W_0 is also called "initial" or "certain" wealth and W is usually referred to as "final" or "terminal" wealth. For what follows we will use these terms interchangeably.