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Decision Support

Assessing financial model risk

Pauline Barrieu[a], Giacomo Scandolo[b,*]

[a] Department of Statistics, London School of Economics, London, UK
[b] Department of Economics, University of Verona, Verona, Italy

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ABSTRACT

Model risk has a huge impact on any risk measurement procedure and its quantification is therefore a crucial step. In this paper, we introduce three quantitative measures of model risk when choosing a particular reference model within a given class: the absolute measure of model risk, the relative measure of model risk and the local measure of model risk. Each of the measures has a specific purpose and so allows for flexibility. We illustrate the various notions by studying some relevant examples, so as to emphasize the practicability and tractability of our approach.

1. Introduction

The specification of a model is a crucial step when measuring financial risks to which a portfolio, or even an institution, is exposed. Common methodologies, such as Delta-Normal or simulation methods, are based on the choice of a particular model for the risk factors, including for instance, equity, interest-rate or credit risk. Even when using historical methods, we implicitly rely on the empirical distribution as the reference model. However, it is observed that the final risk figure is often quite sensitive to the choice of the model. The hazard of working with a potentially not well-suited model is referred to as *model risk*. The study of the impact of model risk and its quantification is an important step in the whole risk measurement procedure. In particular, in the aftermath of the recent financial crisis, understanding model uncertainty when assessing the regulatory capital requirements for financial institutions seems to be crucial. The main goal of this paper is precisely to propose some ways to quantify model risk when measuring financial risks for regulatory purposes. We stress that our objective is *not* to measure risk in the presence of model uncertainty, but to quantify model risk itself.

The question of the impact of model risk has received increasing attention in recent years. In particular, the significance of minimum risk portfolios has been questioned when studying the problem of optimal asset allocation: several authors (among them El Ghaoui, Oks, & Oustry, 2003; Fertis, Baes, & Lüthi, 2012; Zymler, Kuhn, & Rustem, 2013) have recently considered this issue from a robust optimization perspective.

Our approach to assessing model risk is very general. It is based on the specification of a set of alternative models (or distributions) around a reference one. Note that Kerkhof, Melenberg, and Schumacher (2010) propose measuring model risk in a similar setting by computing the worst-case risk measure over a *tolerance set* of models. Our approach differs, however, as we introduce different measures of model risk, based on both the worst- and best-case risk measures, in order to serve different purposes.

Examples of the set of alternative models we can consider include parametric or non-parametric families of distributions, or small perturbations of a given distribution. If we believe in a parametric model, we can consider all distributions within the family whose parameters are in the confidence intervals derived from the data. By doing this, we are accounting only for the *estimation risk* (see Kerkhof et al., 2010). If, on the other hand, we completely believe in some estimated quantities, without relying on confidence intervals, we can consider all possible distributions of any form which are in accordance with those quantities. We can also consider those distributions which are not too far from a reference one, according to some statistical distance (the uniform distance, for instance), or all joint distributions that have the same marginals as the reference one. This latter example leads to the relevant problem of aggregation of risks in a portfolio (see Embrechts, Puccetti, & Rüschendorf, 2013). We could even specify different pricing models if the portfolio contains derivatives.

Note that the scope of our approach is very wide, going beyond issues pertaining just to statistical estimation. Furthermore, the assessment of model risk should not be confused with the analysis of statistical robustness of a risk measurement procedure (as in Cont, Deguest, & Scandolo, 2010), even though the two concepts are related. Indeed, the reference distribution is an input in our approach, while in Cont et al. (2010) it is the result of a statistical estimation process which is part of the definition of robustness itself.

* Corresponding author at: Department of Economics, University of Verona, via dell'Artigliere 19, 37129 Verona, Italy. Tel.: +39 0458028248; fax: +39 0458028529.
E-mail addresses: p.m.barrieu@lse.ac.uk (P. Barrieu), giacomo.scandolo@univr.it (G. Scandolo).

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In order to assess model risk, we introduce three different measures: the absolute measure of model risk, the relative measure of model risk and the local measure of model risk. Our aim is to provide a quantitative measure of the model risk we are exposed to in choosing a particular reference model within a given class when working with a specific risk measure. All three measures are pure numbers, independent from the reference currency. They take non-negative values and vanish precisely when there is no model risk. Each of the measures we propose has a specific purpose and therefore can be used in different contexts: whilst the absolute measure is cardinal and gives a quantitative assessment of model risk, both the relative measure and the local measure are ordinal and allow for comparison of different situations, which may have different scales. If we consider different possible models as references, the use of the relative measure is probably the more natural measure to use as it will give a clear ranking between the alternatives. When the reference model is almost certain, the local measure becomes an obvious choice as it focuses on the very local properties around the reference model.

The measures of model risk we propose can be applied in different contexts. To briefly illustrate this flexibility, we mention here two examples: first, GARCH Value-at-Risk (VaR) procedures and then, computation of Additional Valuation Adjustments (AVA).

Firstly, within the standard GARCH approach for market VaR estimation, the mean $\widehat{\mu}$ and standard deviation $\widehat{\sigma}$ (or volatility) of the returns are estimated using a GARCH model; moreover, the innovations, i.e. the returns standardized by the estimated means and volatilities, are assumed to be normally distributed. Therefore, the VaR is computed under the assumption that returns are normally distributed with parameters $\widehat{\mu}$ and $\widehat{\sigma}$. The model risk that arises from the normality assumption can be quantified using one of the measures presented in this paper. As alternative models, we may consider all distributions (not necessarily normal) with mean $\widehat{\mu}$ and standard deviation $\widehat{\sigma}$.

Secondly, let us consider the recent Capital Requirements Directives (CRD) for the financial services industry, enforced since 2013 in the EU. Under the CRD, financial institutions are required to calculate AVA in all situations in which the market value of an asset is not clear. This should reflect the model risk involved in the valuation of the position. In particular, the last draft of the Regulatory Technical Standard[1] issued by the European Banking Association proposes that: "*institutions shall calculate the model risk AVA by determining a range of plausible valuations produced from alternative appropriate modelling and calibration approaches.*" (Article 11). So, by formally replacing *risk measures* with *valuations* in our framework, the measures of model risk we propose could be effectively used for this purpose.

The rest of the paper is structured as follows. In Section 2, as a motivating example, we show how the *Basel multiplier* can be seen as a rough measure of model risk. In Section 3 we present the definition and the main properties of absolute and relative measures of model risk. Some examples, using alternative sets of distributions based on fixed moments or small perturbations, are provided in Section 4. In Section 5 we discuss local measures of model risk and Section 6 concludes.

2. A motivating example

In this section, we start by looking at the Basel multiplier, introduced by the Basel Committee as an ingredient in the assessment of the capital requirements for financial institutions. As we will see, this multiplier is closely related to probabilistic bounds giving some upper limit to classical risk measures such as the Value-at-Risk and the Expected Shortfall. These preliminary remarks will motivate our approach when introducing some measures for model risk in the next section.

2.1. The Basel multiplier

Within the Basel framework, financial institutions are allowed to use internal models to assess the capital requirement due to market risk. The capital charge is actually the sum of six terms taking into account different facets of market risk. The term that measures risk in *usual* conditions is given by the following formula:

$$\mathrm{CC} = \max\left\{\mathrm{VaR}^{(0)}, \frac{\lambda}{60}\sum_{i=1}^{60}\mathrm{VaR}^{(-i)}\right\}, \tag{1}$$

where $\mathrm{VaR}^{(0)}$ is the portfolio's Value-at-Risk (of order 1 percent and with a 10-day horizon) computed today, while $\mathrm{VaR}^{(-i)}$ is the figure we obtained i days ago.

The constant λ is called the *multiplier* and it is assigned to each institution by the regulator, which periodically revises it. Its minimum value is 3, but it can be increased up to 4 in the event that the risk measurement system provides poor back-testing performances. Given the magnitude of λ, it is apparent that in normal conditions the second term is the leading one in the maximum appearing in (1).

2.2. Chebishev bounds and the multiplier

Stahl (1997) offered a simple theoretical justification for the multiplier to be chosen in the range $[3, 4]$. Here, we briefly summarize his argument. Let X be the random variable (r.v.) describing the Profits-and-Losses of a portfolio due to market risk. If the time-horizon is short, it is usually assumed that $\mathbb{E}[X] = 0$, so that

$$\mathrm{VaR}_\alpha(X) = \sigma\,\mathrm{VaR}_\alpha(\widetilde{X}),$$

where σ^2 is the variance of X and $\widetilde{X} = X/\sigma$ is *standard*, i.e. it has zero mean and unit variance. While σ is a matter of estimation, $\mathrm{VaR}_\alpha(\widetilde{X})$ depends on the assumption we make about the *type* of the distribution of X (normal, Student-t, etc.).

An application of the Chebishev inequality to $\widetilde{X}$ yields

$$P(\widetilde{X} \leqslant -q) \leqslant P(|\widetilde{X}| \geqslant q) \leqslant \frac{1}{q^2}, \quad q > 0. \tag{2}$$

Recalling the definition of VaR, it readily follows $\mathrm{VaR}_\alpha(\widetilde{X}) \leqslant 1/\sqrt{\alpha}$, or

$$\mathrm{VaR}_\alpha(X) \leqslant \frac{\sigma}{\sqrt{\alpha}}. \tag{3}$$

The right hand side of the above inequality thus provides an upper bound for the VaR of a random variable having mean 0 and variance σ^2. It can be compared with the VaR we obtain by using the delta-normal method, which is very commonly employed in practice. According to this method, $\widetilde{X}$ is normally distributed and therefore

$$\mathrm{VaR}_\alpha(X) = \sigma|z_\alpha| \quad (\alpha < 0.5),$$

where $z_\alpha = \Phi^{-1}(\alpha)$ is the quantile of a standard normal. The graph of the ratio

$$\frac{\sigma/\sqrt{\alpha}}{\sigma|z_\alpha|} = \frac{1}{|z_\alpha|\sqrt{\alpha}} \tag{4}$$

is reported below (see Fig. 1, a). We can see that for usual values of α (i.e. from 1 percent to 5 percent), the ratio broadly lies in the interval $[3, 4]$. Therefore, if the VaR computed under normal assumptions is multiplied by λ, we obtain an upper bound for the worst possible VaR compatible with partial information (mean and variance) we have.

We can then extend this argument to the Expected Shortfall.[2] Indeed, by integrating inequality (3), we obtain

$$\mathrm{ES}_\alpha(X) = \frac{1}{\alpha}\int_0^\alpha \mathrm{VaR}_u(X)\,du \leqslant \frac{\sigma}{\alpha}\int_0^\alpha \frac{du}{\sqrt{u}} = \frac{2\sigma}{\sqrt{\alpha}}. \tag{5}$$

[1] Available at www.eba.europa.eu.

[2] Also see Leippold and Vanini (2002).