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Innovative Applications of O.R.

Enhancing non-compensatory composite indicators: A directional proposal

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ABSTRACT

The construction of composite indicators (CI) is useful to synthesize complex social and economic phenomena, but some underlying assumptions in "classical methods", as in particular the compensability among indicators, are very strictly. The aim of this paper is to propose an original approach that enhance non-compensatory issue by introducing "directional" penalties in a Benefit of the Doubt model in order to consider the preference structure among simple indicators. Principal component analysis on simple indicators hyperplane allows to estimate both the direction and the intensity of the average rates of substitution. Under an empirical point of view, our method has been tested on both simulated data and on infrastructural endowment data in European regions.

1. Introduction

Interest in composite indicators (CI) as a tool to support decision-makers in policy analysis context, is rapidly growing thanks to their capability to summarize multi-dimensional issues, to rank countries in benchmarking analysis and to their ease of interpretation.

On the other hand, the construction of a CI is a very complex process with multiple subsequent steps (for a complete explanation of each step, please see Freudenberg, 2003):

1. the systematization of a theoretical framework for the identification of relevant analysis dimensions,
2. the standardization of the simple indicators with the aim of transforming them into pure, dimensionless numbers and to invert possible opposite polarities/signs (*e.g.* air pollution in OECD Better Life Index) in order to allow comparisons,
3. the imputation of missing data,
4. the weighting of simple indicators,
5. the succeeding sensitivity analysis on the robustness of the aggregation.

A critical step of the entire process, focus of our paper, is how to assign unknown weights in order to aggregate simple indicators (Step 4)[1]. In this framework, the two main issues to be considered are: (i) How to find weights *i.e.* if in a subjective or objective manner. (ii) If exists a trade-off relation among simple indicators i.e. the possibility to compensate a disadvantage on some simple indicators with a sufficiently large advantage on the others ones or not.

In order to answer to the first problem, a large number of researchers identify weights subjectively in cooperation with experts who know well the theoretical context (please see *e.g.* ONS, 2002; WMRC, 2001), others, on the contrary, use objective methods (please see Section 2 for details) in order to avoid arbitrariness problems.

The second issue implies taking a position on the fundamental topic of compensability. In fact, a preference relation is compensatory if weights are considered as *intensities* or non-compensatory if weights are considered as *importance coefficients* (please see Munda & Nardo, 2005, 2009; Munda & Saisana, 2011; Munda, 2012a,b, for recent discussion).

From our point of view, opinion-based methods can often introduce distortions in CIs and compensability is not even appropriate in practical applications. For these reasons, in our paper, we propose a weighting method that takes into account an objective non-compensatory preference structure among simple indicators.

The paper is organized as follows. In Section 2 we illustrate a literature review of the common methods for the construction and weighting of composite indicators, in Section 3 we describe our theoretical model, the Section 4 shows two empirical application on simulated data and on infrastructural endowment data in European regions. Conclusion and future perspective are reported in Section 5.

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[1] For seek of simplicity and to focus the attention on this particular step, we do not explain in detail in Section 4.2 the other steps that have been carried out correctly in the standard way.

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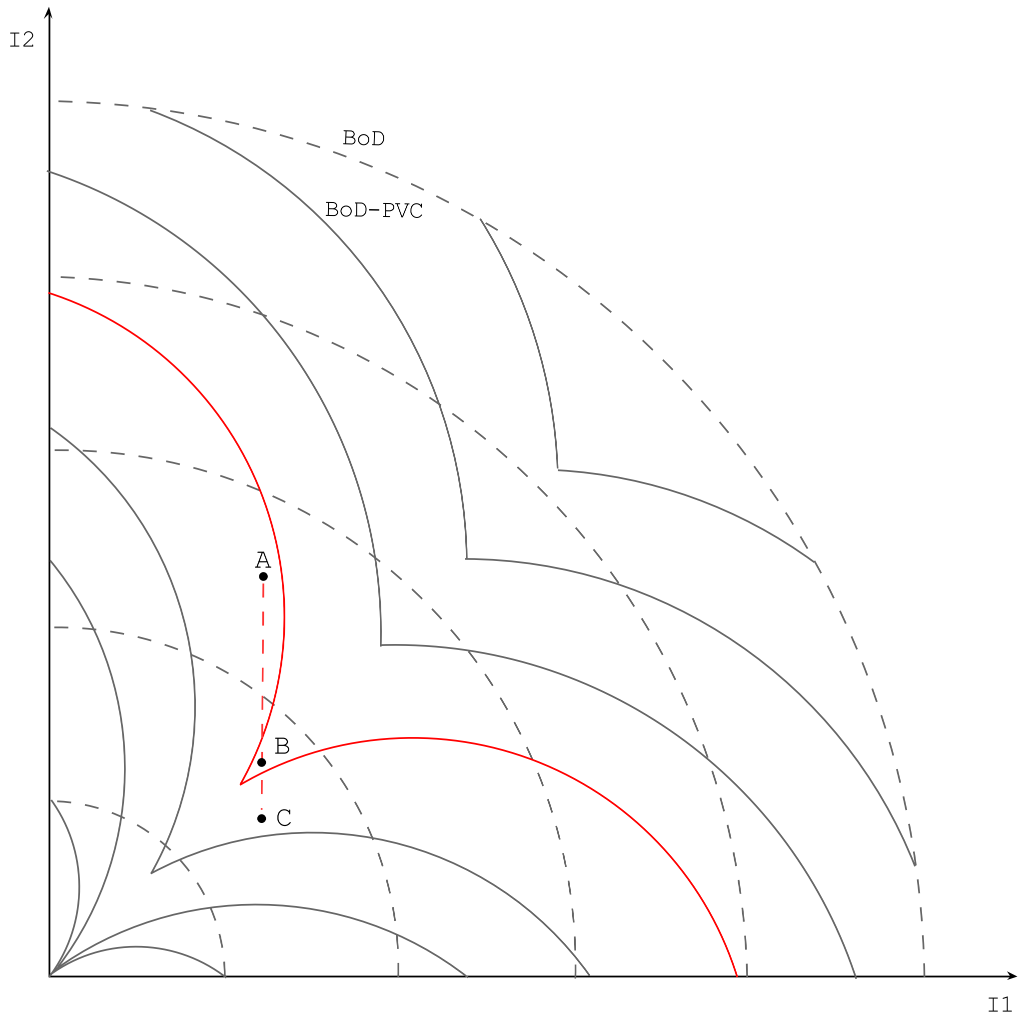

Fig. 1. Comparison between BoD and BoD–PVC.

2. Previous literature

2.1. Aggregating and weighting assumptions for the construction of CIs

In the construction of CIs, the first assumption to make is the functional form for the underlying aggregation rule (see *e.g.* Diewert, 1976) that is generally *linear* (Freudenberg, 2003) i.e. $I = \sum_{i=1}^{N} w_i x_i$, where x_i is a scale adjusted variable normalized in $[0, 1]$ and w_i is the related weight (usually $\sum_{i=1}^{N} w_i = 1, 0 \leq w_i \leq 1$).

This hypothesis is acceptable only under the condition of this theorem: "*given the variables $x_1, x_2, \ldots, x_n$, an additive aggregation function exists if and only if these variables are mutually preferentially independent*" (Debreu, 1960; Keeney & Raiffa, 1993; Krantz, Luce, Suppes, & Tversky, 1971).

Note that a subset of indicators Y is *preferentially independent* of $Y^c = Q$ (the complement of Y) only if any conditional preference among elements of Y, holding all elements of Q fixed, remain the same, regardless of the levels at which Q are held. The variables $x_1, x_2, \ldots, x_n$ are *mutually preferentially independent* if every subset Y of these variables is preferentially independent of its complementary set of evaluators.

Preferential independence is a very strong condition implying the independence between the trade-off ratio of two variables $S_{x,y}$ and the values of the $n - 2$ other variables, i.e. $\frac{\partial S_{x,y}}{\partial q} = 0$, $\forall x, y \in Y, q \in Q$ (Ting, 1971). An additive aggregation function permits the evaluation of the marginal contribution of each variable separately and so the possibility to sum together the single contributions to obtain a total value.

However, in empirical applications often exists collinearity among variables, in this case a linear aggregation could generate biased CIs and so is better to use nonlinear aggregation rules.

Once chosen the aggregation rule, another important assumption in the construction of the CI is the choice of the weighting method. In literature two major fields have been proposed, based on expert subjective judgments or on statistical techniques.

The first group includes budget allocation processes (BAP - Jesinghaus in Moldan, Billharz, & Matravers, 1997) based on a subjective allocation of a "budget" of one hundred points to a set of indicators; analytic hierarchy processes (AHP - Forman, 1983; Saaty, 1987) in which weights are the trade-offs across indicators; conjoint analysis (CA - Green & Srinivasan, 1978; Hair, 1995; McDaniel & Gates, 1998) that studies the evaluations (preferences) given by the respondents on a set of alternative scenarios representing different values for the individual indicators.

In the second group are included: principal component analysis (PCA - Manly, 1994) and factor analysis (FA) that groups collinear simple indicators with the aim to capture the common informations among them; however, weights cannot be estimated with these methods if weak correlation exists among indicators; unobserved components model (UCM - Kaufmann, Kraay, & Zoido-lobatón, 1999; Kaufmann, Kraay, & Mastruzzi, 2003) that assumes the dependence