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Discrete Optimization

A fast algorithm for identifying minimum size instances of the equivalence classes of the Pallet Loading Problem

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ABSTRACT

In this paper, a novel and fast algorithm for identifying the Minimum Size Instance (MSI) of the equivalence class of the Pallet Loading Problem (PLP) is presented. The new algorithm is based on the fact that the PLP instances of the same equivalence class have the property that the aspect ratios of their items belong to an open interval of real numbers. This interval characterises the PLP equivalence classes and is referred to as the Equivalence Ratio Interval (ERI) by authors of this paper. The time complexity of the new algorithm is two polynomial orders lower than that of the best known algorithm. The authors of this paper also suggest that the concept of MSI and its identifying algorithm can be used to transform the non-integer PLP into its equivalent integer MSI.

1. Introduction

The pallet loading problem (PLP) arises in manufacturing workshops (and also in other logistics areas), where small items must be placed onto a large pallet (Dowsland, 1987a, 1987b; Morabito & Morales, 1998; Waescher, Haussner, & Schumann, 2006). The PLP is commonly reduced to its two-dimensional version, in which identical small rectangular items (boxes) must be packed onto a large rectangular pallet; items may be rotated 90°, but only orthogonal packing is allowed. The aim is to pack as many items onto the pallet as possible without overlapping any two items. Using the classification of Dyckhoff (1990) and Waescher et al. (2006), the problem belongs to class 2/B/O/C and IIPP, respectively. This type of PLP problem is also known as the Manufacturer's Pallet Loading Problem (MPLP) (Ribeiro & Lorena, 2007).

The practical significance of the PLP has been expressed by many authors (Alvarez-Valdes, Parreno, & Tamarit, 2005; Birgin, Lobato, & Morabito, 2010; Dowsland, 1987a; Dyckhoff, 1990; Martins & Dell, 2008; Nelissen, 1993; Ram, 1992; Young-Gun & Maing-Kyu, 2001) in the course of solving the PLP. In these publications, the authors state or imply that the only constraint is the stability and safety of the boxes and thus use orthogonal placement (e.g., Dowsland, 1987a; Martins & Dell, 2008; Nelissen, 1993; Young-Gun & Maing-Kyu, 2001). There are also a small number of authors who consider additional constraints, such as clampability (e.g., Carpenter & Dowsland, 1985). Recently, Kocjan and Holmström (2010) deals with specific loads of PLP, and Bortfeldt and Wäscher (2013) reviews special constraints on general container loading. The PLP also occurs in a number of cutting stock and floor design scenarios (Martins & Dell, 2008).

The PLP is difficult to solve exactly within a short computation time, and sample instances are often used to demonstrate an algorithm's efficiency (Martins, 2003; Martins & Dell, 2007). Dowsland (1984) first showed that PLP instances can be divided into equivalence classes with the same optimal placement pattern. Such authors as Dowsland (1984, 1987a, 1987b), Scheithauer and Terno (1996), Nelissen (1993), Morabito and Morales (1998), and Pureza and Morabito (2006) used PLP instances of equivalent classes numbering from approximately 10,000 to approximately 50,000. Restrictions on the aspect ratio of the pallet and the items are commonly applied to confine the number of instances investigated. Alvarez-Valdes et al. (2005), Birgin, Morabito, and Nishihara (2005), and Lins, Lins, and Morabito (2003) investigated a common set of approximately 50,000 instances.

Martins and Dell (2007) first proposed the idea of identifying a PLP equivalence class by the Minimum Size Instance (MSI). The input of a PLP instance contains four parameters: the length and width of the pallet and the length and width of the boxes. An MSI of an equivalent class is the equivalent instance that has all four parameters minimised. Martins and Dell (2007) proved that the existence and uniqueness of an MSI for a PLP equivalence class is guaranteed. These researchers also found the parameter bounds of the MSI when the PLP's area ratio (the ratio of pallet area to item

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area) is bounded. Thus, an enumerating procedure can be designed to generate all equivalence classes of a PLP subject to a given area ratio bound.

According to Martins and Dell (2007), the motivation for determining the MSI is to identify equivalent classes by unique instances and to provide a systematic way of exhaustively enumerating PLP equivalent classes subject to a given boundary (such as the area ratio bound). It should be noted that Dowsland (1984) also demonstrates a method for enumerating PLP equivalent classes subject to given bounding conditions, and Dowsland (1987a) also mentions minimising the smaller dimension of the box such that the solution space is smaller and easier to solve. However, Martin and Dell's work clarifies the concept of MSI and provides a systematic way to use it.

Exhaustively enumerating PLP equivalence classes also provides an exact solution scheme that reuses MSI solutions. Because most PLP equivalent classes are easily solvable, the hard classes may be tackled at the cost of a long run time or expensive hardware. However, once the classes are solved, the solutions, represented by the solutions of MSIs, can be stored in a database for future reference. Both Dowsland (1987b) and Martins and Dell (2008) suggest using this scheme for completely solving the PLP subject to a given area ratio bound.

The MSI enumeration algorithms presented by Martins and Dell (2007) are of a high-order polynomial. Enumerating all MSIs of area ratio $N < 101$ requires the maximum bearable computing load on a 600-Megahertz computer (with a 2.8-Gigahertz computer, $N < 151$ is bearable).

The input sizes of sample instances for demonstrating PLP algorithms continue to grow over time. The Cover I dataset has an area ratio of $N < 51$ and was used by the earliest authors (Dowsland, 1984, 1987a). Later, a dataset with $N < 101$ (the Cover II dataset) was used by most authors until 2010. In 2010, Birgin et al. (2010) used a Cover III dataset with $N < 151$. To the best of the author's knowledge, $N < 151$ is currently the largest input size being used in PLP datasets.

Although the Cover I, II, and III datasets are valuable for developing algorithms, they are far from representing the requirements of real applications. For example, the practical wood pulp stowage problem (Birgin et al., 2010) involves area ratios ranging from 147 to 341. The area ratio of the PLP in container loading may reach values in the several thousands (Young-Gun & Maing-Kyu, 2001).

The L-shaped heuristic is a notably effective PLP algorithm and is estimated to be exact (Birgin et al., 2005; Lins et al., 2003). This algorithm has been tested by Cover I ($N < 51$), Cover II ($N < 101$), and Cover III ($N < 151$) but has not been systemically tested by equivalent classes with $N \geqslant 151$.

In this paper, the authors introduce a new way for identifying the MSI. The new MSI-identifying algorithm's computing complexity is two polynomial orders lower than that of the algorithm of Martins and Dell (2007). With a 2.8-Gigahertz computer, the new algorithm enumerates all of the equivalence classes with an area ratio $N < 101$ in approximately two minutes. All equivalence classes with an area ratio $N < 201$ and $N < 301$ are enumerated. Additionally, the new algorithm is based on an item aspect ratio interval (equivalence ratio interval), which provides a new perspective on the nature of equivalence classes and the MSI. The authors of this paper also note that the concept of MSI and its identifying algorithm can be used to transform a non-integer PLP into its equivalent integer MSI. The new algorithm is simple to implement. When programmed in C++, the total code length of the equivalence class enumerating algorithm is only 127 lines.

The following are suggested applications of the algorithm described in this paper:

1. As a subroutine of the exact solution scheme that reuses MSI solutions (proposed by Martins & Dell, 2008; Dowsland, 1987b), the MSI-identifying algorithm discussed herein, which has $O(N^2)$ complexity, can be used to substitute the corresponding Martins and Dell (2008) MSI-identifying algorithm, which has $O(N^4)$ complexity.
2. The $N < 201$ and the $N < 301$ equivalent class datasets can be used for algorithm testing or demonstration and can also be used to expand the capacity of the exact solution scheme that reuses MSI solutions.
3. The 127 C++ code lines, which are much easier to distribute and execute much more rapidly, can be used to substitute for the previous $N < 151$ datasets, which are difficult to describe in publication materials or share online. Of course, the code can also serve as the source of the $N < 201$ and $N < 301$ datasets.
4. For a dataset that has $N \geqslant 301$, because the number of instances will grow to be excessively large, the authors suggest using the 127 C++ code lines plus additional constraints and/or selection mechanisms, such as the uniform random selection, to generate sample instances for the purpose of algorithm testing and demonstration.
5. The MSI identification algorithm can be employed to convert non-integer PLP instances from real applications to their equivalent integer MSIs without losing any precision such that they can be solved using up-to-date algorithms that involve integer assumptions.
6. As pointed out by Dowsland (1985, 1987a), the MSI reduces the problem to one with fewer potential placement positions, and usually strengthens bounds such as that due to Barnes, thus improving the efficiency of many exact approaches to the problem. This can be done only when we have a high efficiency MSI-identifying algorithm, especially when the area ratio of the problem is large. Thus the $O(N^2)$ MSI-identifying algorithm presented in this paper is better than the previous $O(N^4)$ algorithm for this purpose.

Presently, the $O(N^4)$ algorithm by Martins and Dell (2008) is the only well described MSI-identifying algorithm. In Dowsland (1987a), it is stated that using an idea similar to the aspect ratio interval to minimise the smaller dimension of the box could reduce the search space. However, the detailed procedure is not described. In this paper we will present the $O(N^2)$ MSI-identifying algorithm with theoretic details, the computational analysis, and the code realisation along with a numerical test.

The rest of this paper will be organised as follows. In Sections 2.1 and 2.2, basic notation, assumptions, and commonly known properties of the PLP are presented or reviewed. In Section 2.3, the general scheme of the new method is concisely introduced. Details of the new method are introduced in Sections 3–5. Section 6 addresses the implementation issue, where an integer realisation is proposed to avoid the precision issue associated with float computing. Section 7 provides numerical results, and Section 8 concludes the paper.

2. Notations, properties, and the general scheme

2.1. Notations and assumptions

In this paper, a PLP instance is represented by (X, Y, a, b). X and Y are the pallet's width and height; a and b are the item's width and