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The Red–Blue transportation problem

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ABSTRACT

This paper considers the Red–Blue Transportation Problem (Red–Blue TP), a generalization of the transportation problem where supply nodes are partitioned into two sets and so-called exclusionary constraints are imposed. We encountered a special case of this problem in a hospital context, where patients need to be assigned to rooms. We establish the problem's complexity, and we compare two integer programming formulations. Furthermore, a maximization variant of Red–Blue TP is presented, for which we propose a constant-factor approximation algorithm. We conclude with a computational study on the performance of the integer programming formulations and the approximation algorithms, by varying the problem size, the partitioning of the supply nodes, and the density of the problem.

1. Introduction

Consider the well-known Transportation Problem (TP): given is a set of supply nodes S, each with supply a_i $(i \in S)$, a set of demand nodes D, each with demand b_j $(j \in D)$, with $\sum_{i \in S} a_i = \sum_{j \in D} b_j$, and a bipartite graph $(S \cup D, E)$, with a given cost c_{ij} for each edge $(i,j) \in E$, where E is not necessarily complete. The question is how to send the flow from supply nodes to the demand nodes such that total cost is minimal. In this paper, we generalize this problem by associating a color, either red or blue, to each supply node. Thus, the set of supply nodes is partitioned into two sets R (red) and B (blue) such that $S = R \cup B$, and $R \cap B = \emptyset$. The additional requirement is that the set of supply nodes that actually supply a demand node should all have the same color. In other words, a demand node is only allowed to receive flow from supply nodes that are either all red or all blue. We refer to these constraints as color constraints. Obviously, the resulting problem is a generalization of the transportation problem since if all supply nodes have the same color, the TP arises. We will refer to our problem as the Red–Blue Transportation Problem (Red–Blue TP).

In Section 1.1 we discuss the practical application that motivated our study, followed by related literature in Section 1.2.

1.1. Motivation

Although the Red–Blue TP may seem a purely theoretical generalization of the transportation problem, its motivation stems from a situation we encountered in practice. Consider a setting where patients in a hospital need to be assigned to rooms. Rooms are of limited capacity, and due to specific equipment, not all rooms are equally appropriate for each patient. For instance, a patient's pathology may require oxygen to be available at the room; rooms that do not meet this requirement, need to be equipped with a mobile oxygen supply which is, for organizational reasons, less desirable. The Patient Admission Scheduling problem (PAS) consists in assigning patients to rooms in such a way that the medical concerns and personal wishes are fulfilled as much as possible. This problem has been defined by Demeester, Souffriau, De Causmaecker, and Vanden Berghe (2010), and studied further by Ceschia and Schaerf (2011). A key constraint in the patient admission scheduling problem is that male and female patients should not be assigned to the same room, which is common practice in hospitals (all over the world). Clearly, this situation can be modeled as a (special case of) Red–Blue TP: each patient is represented as a supply node with $a_i = 1$, each room is represented as a demand node where the capacity of the room is represented by b_j, and the "appropriateness" of assigning patient i to room j is captured by cost c_{ij}.

It is not hard to think of other practical applications of Red–Blue TP. For instance, imagine a situation where a number of goods need to be transported from a port to a warehouse. Several trucks are

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available for transportation, each driving according to a schedule that fixes the departure times at the port. Depending on delivery deadlines, some truck assignments are more suitable for particular goods than others. A Red–Blue TP instance arises if the goods can be divided into two types that cannot be assigned to the same truck, for instance because of incompatibilities of content (e.g. hazardous materials), ownership (e.g. rivaling business companies that are unwilling to have their goods transported on the same truck), or size (Cao & Uebe, 1995). Another application involves the transportation of football fans to the match using public railways: when assigning fans to trains, no fans of the opposing teams should be on the same train, to avoid hooliganism (Schreuder, 1992). In Section 4, we mention further applications of a maximization variant of our problem.

1.2. Related literature

The Red–Blue transportation problem is a natural generalization of a classic problem in operations research. In the literature, several generalizations of the transportation problem have been described. The most well-known is probably the transshipment problem, in which the underlying graph need not be bipartite, and so-called *transferring* nodes, which have no net supply or demand, may exist (see e.g. Orden (1956)). The min-cost flow problem is a further generalization of the transshipment problem, introducing capacities on the arcs. In the fixed-charge transportation problem (Hirsch & Dantzig, 1968), a fixed cost may be incurred for every arc in the transportation network that is used. Numerous other generalizations of the transportation problem have been presented, for instance to solve spatial economic equilibrium problems (MacKinnon, 1975), and aircraft routing problems (Ferguson & Dantzig, 1955), or even to deal with wartime conditions where distances from some sources to some destinations are no longer definite (i.e. the *gray* transportation problem, see Bai, Mao, & Lu (2004)).

One generalized transportation problem is particularly related to the Red–Blue transportation problem, namely the Transportation Problem with Exclusionary Side Constraints (TPESC). Although the name TPESC was coined by Sun (2002), it was in fact introduced by Cao (1992). The phenomenon that not every set of supply nodes is allowed to send flow to a demand node, is something that TPESC and Red–Blue TP have in common. In TPESC, for each demand node $j \in D$, a set of pairs of supply nodes is given, denoted by $F_j = \{\{i_1, i_2\} | i_1, i_2 \in S\}$. The problem is to send the flow from supply to demand nodes at minimum cost, such that each demand node $j \in D$ only receives supply from at most one supply node for each pair of supply nodes present in F_j.

It is not hard to see that Red–Blue TP is a special case of TPESC. Goossens and Spieksma (2009) show that TPESC is NP-hard, and becomes pseudo-polynomially solvable if the number of supply nodes is fixed. Furthermore, these authors study TPESC with identical exclusionary sets: they provide a pseudo-polynomial algorithm for the case with two demand nodes, and prove NP-hardness for the case with three demand nodes.

Another problem related to Red–Blue TP is the so-called Maximum Flow problem with Conflict Graph (MFCG), a problem studied by Pferschy and Schauer (2013). In the MFCG a directed graph with capacitated arcs, a source, and a sink are given. In addition, pairs of arcs (from the directed graph) are given; for some pairs of arcs the constraint is that at most one arc of the pair can carry flow (a negative disjunctive constraint), for other pairs of arcs the constraint is that at least one arc of the pair must carry flow (a positive disjunctive constraint). Pferschy and Schauer (2013) show that the problem of finding a maximum flow in a network under these disjunctive constraints is (strongly) NP-hard; even more they show that no polynomial time constant-factor approximation algorithm can exist (unless P = NP).

Observe that Red–Blue TP is a special case of MFCG; indeed, consider some demand $j \in D$. Now, by having negative disjunctive constraints for each pair of arcs that consist of one arc emanating from a red supply node to node j, and one arc emanating from a blue supply node to node j, an instance of Red–Blue TP arises. We point out that for our special case it is possible to find polynomial time constant factor approximation algorithms (see Section 4).

2. Complexity of Red–Blue TP

As a general statement of the complexity of Red–Blue TP, we provide the following theorem.

Theorem 1. *Red–Blue TP is NP-hard, even if* $a_i = 1\ \forall i \in S$, *and* $b_j = 3\ \forall j \in D$.

Proof. We prove Theorem 1 by showing that the EXACT-3-COVER (X3C) problem can be reduced to the decision version of Red–Blue TP. The decision version of Red–Blue TP, denoted Red–Blue TP_D, concerns the question: does there exist a solution that sends all flow from the supply nodes to the demand nodes while satisfying demand, and while satisfying the color constraints, i.e. does there exist a feasible flow? X3C has been shown to be NP-complete (see e.g., Garey & Johnson, 1979), and is defined as follows:

> **Input:** A set X with $|X| = 3q$ and a collection C of 3-element subsets (i.e., triples) of X, with $|C| = k$.
> **Question:** Does there exist a cover in C that covers X exactly, i.e. a subcollection $C\prime \subseteq C$ such that every $x_i \in X$ is contained in exactly one $C_j \in C'$?

Any instance of X3C (with $|C| > q$) can be reduced to Red–Blue TP_D as follows. Associate to each element $x_i \in X$ a blue supply node i with $a_i = 1$. Associate to each triple C_j a demand node j with $b_j = 3$. Create edges from supply to demand nodes corresponding to the membership relations (i.e. supply node x_i is connected to demand node $C_j \iff x_i \in C_j$). Add $3(k - q)$ red supply nodes with $a_i = 1$ that are connected to all demand nodes. Observe that total supply equals total demand. The question is: does there exist a feasible flow in this instance of Red–Blue TP_D?

Now we show that a yes-answer to the X3C instance directly corresponds to a yes-answer to the corresponding Red–Blue TP instance, and vice versa.

First, consider an X3C instance that is feasible, and thus has an exact cover $C' \subseteq C$. Then, each demand node corresponding to a $C_j \in C'$ can be supplied by the blue supply nodes corresponding to the $x_i \in C_j$, and the remaining demand nodes can be supplied by the red supply nodes. Thus, the corresponding Red–Blue TP_D instance is also feasible.

Next, consider any feasible solution to the Red–Blue TP_D instance. Each demand node is supplied by either three red supply nodes or by three blue supply nodes. Moreover, there must exist q demand nodes each supplied by three blue supply nodes. These triples of blue supply nodes correspond to the triples in X3C that form a feasible solution. □

Notice that the above reduction can be generalized to show that Red–Blue TP with $b_j = k$ is at least as hard as Exact Cover by k-sets.

If we put a cost of zero on the edges described in the above proof, and add some edges with a cost strictly larger than zero (corresponding to $x_i \notin C_j$), a polynomial-time algorithm with a constant performance ratio for Red–Blue TP would find a zero cost solution if one exists, and hence would be able to distinguish between the yes-instances and the no-instances of X3C. Therefore, the following corollary holds: