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Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade credit financing

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ABSTRACT

In a supplier-retailer-buyer supply chain, the supplier frequently offers the retailer a trade credit of *S* periods, and the retailer in turn provides a trade credit of *R* periods to her/his buyer to stimulate sales and reduce inventory. From the seller's perspective, granting trade credit increases sales and revenue but also increases opportunity cost (i.e., the capital opportunity loss during credit period) and default risk (i.e., the percentage that the buyer will not be able to pay off her/his debt obligations). Hence, how to determine credit period is increasingly recognized as an important strategy to increase seller's profitability. Also, many products such as fruits, vegetables, high-tech products, pharmaceuticals, and volatile liquids not only deteriorate continuously due to evaporation, obsolescence and spoilage but also have their expiration dates. However, only a few researchers take the expiration date of a deteriorating item into consideration. This paper proposes an economic order quantity model for the retailer where: (a) the supplier provides an up-stream trade credit and the retailer also offers a down-stream trade credit, (b) the retailer's down-stream trade credit to the buyer not only increases sales and revenue but also opportunity cost and default risk, and (c) deteriorating items not only deteriorate continuously but also have their expiration dates. We then show that the retailer's optimal credit period and cycle time not only exist but also are unique. Furthermore, we discuss several special cases including for non-deteriorating items. Finally, we run some numerical examples to illustrate the problem and provide managerial insights.

Introduction

In practice, the seller usually provides to her/his buyer a permissible delay in payments to stimulate sales and reduce inventory. During the credit period, the buyer can accumulate the revenue and earn interest on the accumulative revenue. However, if the buyer cannot pay off the purchase amount during the credit period then the seller charges to the buyer interest on the unpaid balance. One of the first works along this line of research is Goyal (1985). He established the retailer's optimal economic order quantity (EOQ) when the supplier offers a permissible delay in payments. On the other hand, Shah (1993) then considered a stochastic inventory model for deteriorating items when delays in payments are permissible. Later, Aggarwal and Jaggi (1995) extended the EOQ model from non-deteriorating items to deteriorating items. Jamal, Sarker, and Wang (1997) further generalized the EOQ model with trade credit financing to allow shortages. After, Teng (2002) provided an easy analytical closed-form solution to this type of problem. Afterwards, Huang (2003) extended the trade credit problem to the case in which a supplier offers its retailer a credit period, and the retailer in turn provides another credit period to its customers. Furthermore, Liao (2008) extended Huang's model to an economic production quantity (EPQ) model for deteriorating items. Subsequently, Teng (2009) provided the optimal ordering policies for a retailer to deal with bad credit customers as well as good credit customers. Conversely, Min, Zhou, and Zhao (2010) proposed an EPQ model under stock-dependent demand and two-level trade credit. Later, Kreng and Tan (2011) obtained the optimal replenishment decision in an EPQ model with defective items under trade credit policy. After, Teng, Krommyda, Skouri, and Lou (2011) obtained the optimal ordering policy for stock-dependent demand under progressive payment scheme. Further, Teng, Min, and Pan (2012) extended the demand pattern from

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constant to increasing in time. Recently, Ouyang and Chang (2013) built up an EPQ model with imperfect production process and complete backlogging. Concurrently, Chen, Cárdenas-Barrón, and Teng (in press) established the retailer's optimal EOQ when the supplier offers conditionally permissible delay in payments link to order quantity. In all articles described above, the EOQ/EPQ models with trade credit financing were studied only from the perspective of the buyer. How to determine the optimal credit period for the seller has received only a few attentions by the researchers such as Chern, Pan, Teng, Chan, and Chen (2013) and Teng and Lou (2012). Currently, Seifert, Seifert, and Protopappa-Sieke (2013) organized the trade credit literature and derived a detailed agenda for future research in trade credit area.

It is well know that many products such as vegetables, fruits, volatile liquids, blood banks, fashion merchandises and high-tech products deteriorate continuously due to several reasons such as evaporation, spoilage, obsolescence among others. In this course, Ghare and Schrader (1963) proposed an EOQ model by assuming an exponentially decaying inventory. Then Covert and Philip (1973) generalized the constant exponential deterioration rate to a two-parameter Weibull distribution. Later, Dave and Patel (1981) established an EOQ model for deteriorating items with linearly increasing demand and no shortages. Then Sachan (1984) further extended the EOQ model to allow for shortages. Conversely, Goswami and Chaudhuri (1991) generalized an EOQ model for deteriorating items from a constant demand pattern to a linear trend in demand. Concurrently, Raafat (1991) provided a survey of literature on continuously deteriorating inventory model. On the other hand, Hariga (1996) studied optimal EOQ models for deteriorating items with time-varying demand. Afterwards, Teng, Chern, Yang, and Wang (1999) generalized EOQ models with shortages and fluctuating demand. Later, Goyal and Giri (2001) wrote a survey on the recent trends in modeling of deteriorating inventory. Teng, Chang, Dye, and Hung (2002) further extended the model to allow for partial backlogging. Skouri, Konstantaras, Papachristos, and Ganas (2009) established inventory EOQ models with ramp-type demand rate and Weibull deterioration rate. In a subsequent paper, Skouri, Konstantaras, Papachristos, and Teng (2011) further generalized the model for deteriorating items with ramp-type demand and permissible delay in payments. Mahata (2012) proposed an EPQ model for deteriorating items under retailer partial trade credit policy. Recently, Dye (2013) studied the effect of technology investment on deteriorating items. Wee and Widyadana (2013) developed a production model for deteriorating items with stochastic preventive maintenance time and rework. Although a deteriorating item has its own expiration date (a.k.a., maximum lifetime), none of the above mentioned papers take the maximum lifetime into consideration. Currently, Bakker, Riezebos, and Teunter (2012) wrote a review of inventory systems with deterioration since 2001.

In this paper, we propose an EOQ model for the retailer to obtain her/his optimal credit period and cycle time when: (a) the supplier grants to the retailer an up-stream trade credit of S years while the retailer offers a down-stream trade credit of R years to the buyer, (b) the retailer's down-stream trade credit to the buyer not only increases sales and revenue but also opportunity cost and default risk, and (c) a deteriorating item not only deteriorates continuously but also has its maximum lifetime. We then formulate the retailer's objective functions under different possible cases. In fact, the proposed inventory model forms a general framework that includes many previous models as special cases such as Goyal (1985),eng (2002), Teng and Goyal (2007), Teng and Lou (2012), Lou and Wang (2013), Wang, Teng, and Lou (2014), and others. By applying concave fractional programming, we prove that there exists a unique global optimal solution to the retailer's replenishment cycle time. Similarly, using Calculus we show that the retailer's optimal down-stream credit period not only exists but also is unique. Furthermore, we discuss a special case for non-deteriorating items. Finally, we run several numerical examples to illustrate the problem and provide some managerial insights.

The rest of the paper is designed as follows. To establish the proposed EOQ model, we define the notation and assumptions in section 'Notation and assumptions'. Then we derive mathematical expressions of the relevant factors and the retailer's annual total profit function under each distinct possible case in section 'Mathematical model'. In section 'Theoretical results and optimal solution', we show that both the optimal cycle time and the optimal trade credit exist uniquely by applying concave fractional programming and Calculus, respectively. In section 'Some special cases', several previous EOQ models with trade credit financing are shown to be special cases of the proposed model including those non-deteriorating items. Numerical examples and sensitivity analysis are presented to illustrate the model and provide managerial insights in section 'Numerical examples'. Finally, the conclusion and future extensions of the proposed model are established in section 'Conclusions and future research'.

Notation and assumptions

The following notation and assumptions are used in the entire paper.

Notation

For the retailer

o	ordering cost per order in dollars.
c	purchase cost per unit in dollars.
p	selling price per unit in dollars, with $p > c$.
h	unit holding cost per year in dollars excluding interest charge.
r	annual compound interest paid per dollar per year.
I_e	interest earned per dollar per year.
I_c	interest charged per dollar per year.
t	the time in years.
$I(t)$	inventory level in units at time t.
$\theta(t)$	the time-varying deterioration rate at time t, where $0 \leqslant \theta(t) < 1$.
m	the expiration date or maximum lifetime in years of the deteriorating item.
S	up-stream credit period in years offered by the supplier.
R	down-stream trade credit period in years offered by the retailer (a decision variable).
$D = D(R)$	the market annual demand rate in units which is a concave and increasing function of R.
T	replenishment cycle time in years (a decision variable).
Q	order quantity.
$TP(R,T)$	total annual profit, which is a function of R and T.
R^*	optimal down-stream credit period in years.
T^*	optimal replenishment cycle time in years.
TP^*	optimal annual total profit in dollars.

Assumptions

Next, the following assumptions are made to establish the mathematical inventory model.

1. All deteriorating items have their expiration dates. Hence, the deterioration rate must be closed to 1 when time is approaching to the expiration date m. We may assume that the deteriorating