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Decision Support

Developing real option game models

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ABSTRACT

By mixing concepts from both game theoretic analysis and real options theory, an investment decision in a competitive market can be seen as a "game" between firms, as firms implicitly take into account other firms' reactions to their own investment actions. We review two decades of real option game models, suggesting which critical problems have been "solved" by considering game theory, and which significant problems have not been yet adequately addressed. We provide some insights on the plausible empirical applications, or shortfalls in applications to date, and suggest some promising avenues for future research.

1. Introduction

Investment in competitive markets is a "game" among firms, since in making investment decisions, firms implicitly take into account what they think will be the other firms' reactions to their own investment actions, and realize that their competitors think the same way. Consequently, as game theory aims to provide an abstract framework for modeling situations involving interdependent choices, and "real options" ("RO") theory is appropriate for most investment decisions, a combination of these two theories have yielded promising results.

A "standard real option game" ("SROG") model is where the value of the investment is treated as a state variable that follows a known process[1]; time is considered infinite and continuous; the investment cost is sunk, indivisible and known[2]; firms are not financially constrained; the investment problem is studied in isolation as if it is the only asset on the firm's balance sheet (i.e., the game is played on a single project); and there are usually two firms holding the option to invest.[3] The focus is the derivation of the firms' value functions and their respective investment thresholds, under the assumption that either firms are risk-neutral or the stochastic evolution of the variable(s) underlying the investment value is spanned by the current instantaneous returns from a portfolio of securities that can be traded continuously without transaction costs in a perfectly competitive capital market.

The two most common investment games are the "pre-emption game" (PE) and the "war-of-attrition game" (WOA), both usually formulated as non-zero-sum games (i.e. firms can improve their profits without eliminating the profits of rivals). In the PE, it is assumed that there is a "first-mover advantage" ("FMA") that gives firms an incentive to be the first to invest. In the attrition game, it is assumed that there is a second-mover advantage that gives firms an incentive to be the second to invest. Typically the advantage of investing first/second is assumed to be limited,[4] so the investment of the leader (PE) or the follower (WOA) does not completely eliminate the revenues of its rival. The investment game is treated as a "one-shot" game (i.e., firms are allowed to invest only once); firms invest either sequentially or simultaneously, or both; cooperation between firms is not allowed; the market for the project, underlying the investment decision, is considered to be complete and frictionless; and firms are assumed to be ex-ante (i.e., before the investment) and ex-post (i.e., after the investment) symmetric.

In models where the RO value is driven by just one stochastic underlying variable, the firm's optimal investment timing is defined by a point; in models that use two stochastic underlying variables, by a line; and in models that use three or more stochastic underlying variables, by a surface or other more complex space structures. However, regardless of the number of underlying variables used, the principle remains the same: "a firm should

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[1] Typically, geometric Brownian motion (gBm) and mean reverting processes, stochastic processes with jumps, birth and death processes, or combinations of these processes.

[2] Some authors relax this assumption, such as Dixit and Pindyck (1994, chap. 6) and Azevedo and Paxson (2011) where the investment cost follows a gBm process.

[3] See Bouis, Huisman, and Kort (2009) for an example of a RO model with three firms.

[4] Exceptions to this rule are Williams (1993) and Murto and Keppo (2002) models, derived for a context of complete pre-emption.

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invest as soon as its investment threshold is crossed the first time". "Non-standard real options games" ("NSROG") relax some of these assumptions and constraints.[5]

The three most basic elements that characterize a game are the players, their strategies and payoffs. Translating these to a "real option game" ("ROG"), the players are the firms that hold the option to invest, the strategies are the choices "invest"/"defer" and the payoffs are the firms' value functions. Additionally, to be fully characterized, a game still needs to be specified in terms of what sort of information (complete/incomplete, perfect/imperfect, symmetric/asymmetric) the players have at each instant. Also required are what type of game is being played (a "one-shot" game, a "zero-sum" game, a sequential/simultaneous game, or a cooperative/non-cooperative game); and whether mixed strategies are allowed.

One difference between a "standard game" ("SG") from game theory and a SROG is in the way the players' payoffs are given. In SG such as the "prisoners' dilemma", the "grab-the-dollar", the "burning the bridge" or the "battle-of-the-sexes", the players' payoffs are usually deterministic, while in SROG they are given by sometimes complex mathematical functions that depend on one, or more, stochastic underlying variables.[6] This fact changes radically the rules under which the game equilibrium is determined, because if the players' payoffs depend on time, and time is continuous, the game is played in continuous-time. But, if the game is played in a continuous-time and players can move at any time, what does the strategy "move immediately after" mean? In the RO literature, the approach used to overcome this problem is usually based on Fudenberg and Tirole (1985) – F&T (1985) – who develop a new formalism for modeling deterministic timing games and introduce the "principle of rent equalization" for pre-emption games, a methodology which was extended to stochastic ROG.[7]

The main principle underlying game theory is that those involved in strategic decisions are affected not only by their own choices but also by the decisions of others. Once the structure of a game and the strategies of the players are set, the equilibrium of the game can be determined using Nash (1950, 1953).[8]

We cite an extensive number of papers, published or in progress, modeling investment decisions considering uncertainty and competition. Our goal is to organize two decades of literature on ROG models into "game-theoretic categories", a unique contribution in the literature, giving particular emphasis to the models underlying game-theoretic aspects in terms of what has been accomplished, relating the accomplished results to the known empirical evidence and industry applications, if any.[9] There is a consensus among researchers that it might be possible to develop more sophisticated ROG through a better integration between real options and game theory. We suggest new avenues for future research, partly based on perceived (game-theoretic related) gaps in the literature.[10]

This paper is organized as follows. In Section 2, we introduce basic aspects of the SROG models, discuss the mathematical formulation, principles and methodologies commonly used to derive the firms' value functions (payoffs) and investment thresholds, and introduce and contrast the discrete-time and the continuous-time frameworks when applied to ROG. Section 3 reviews two decades of academic research on "standard" and "non standard" ROG. Section 4 surveys the limited empirical research and suggests some testable hypotheses. Section 5 concludes and suggests new avenues for research.

2. SROG framework

Consider an industry comprised of two (ex-ante/ex-post) identical firms, possessing an option to invest in the same (and unique) project that will produce a unit of output. The irreversible investment cost is I and the cash flow stream from the investment is uncertain. The payoff of each firm is affected by the actions (strategy) of its rival.[11] The price unit of output, $P(t)$ fluctuates stochastically over time according to Eq. (1),

$$P(t) = X(t)D[Q(t)] \tag{1}$$

where $D[Q(t)]$ is the inverse demand function, with $Q(t)$ representing the industry supply process. The market supply has three states, $Q(t) = 0$, $Q(t) = 1$ and $Q(t) = 2$, for the scenarios where both firms are idle, only the leader is active and both firms are active, respectively. The inverse demand function is downward sloping ($D'[Q(t)] < 0$), which ensures a FMA; and $X(t)$ is an exogenous shock process to demand following a gBm process given by Eq. (2).

$$dX = \mu_X X\,dt + \sigma_X X\,dz \tag{2}$$

where μ_X is the instantaneous conditional expected percentage change in X per unit of time; σ_X is the instantaneous conditional standard deviation per unit of time; and dz is the increment of a standard Wiener process for the variable X.

At the beginning of the investment game each firm contemplates two choices, whether it should be the first to exercise the option (becoming the leader) or the second to exercise (entering the market as a follower), having for each of these strategies an optimal time to act. The equilibrium set of exercise strategies is derived by letting the firms choose their roles and deriving the value functions of both firms, starting with the follower's value function and then working backwards in a dynamic programming fashion to determine the leader's value function.

Denoting $F_F(X)$ as the value of the follower before investing and assuming that firms are risk-neutral, $F_F(X)$ must solve the following equilibrium differential equation:

$$\frac{1}{2}\sigma_X^2 X^2 \frac{\partial^2 F_F(X)}{\partial X^2} + \mu_X X \frac{\partial F_F(X)}{\partial X} - rF_F(X) = 0 \tag{3}$$

The differential Eq. (3) must be solved subject to the boundary conditions (4) and (5), which ensure that the follower invests at a moment where its "option to invest" is maximized:

$$F_F(X_F^*) = \frac{X_F^* D(2)}{r - \mu_x} - I \tag{4}$$

$$F_F'(X_F^*) = \frac{D(2)}{r - \mu_x} \tag{5}$$

where X_F^* is the value of $X(t)$ that triggers entry. For convergence of the solution we assume that the asset yield $\delta_x = (\mu_X - r) > 0$, where r

[5] Models that use more than one stochastic underlying variable are defined here as NSROG.

[6] In SROG the game "payoffs" are denoted "value functions".

[7] Sometimes without a proper consideration of its appropriateness, as highlighted by Thijssen, Huisman, and Kort (2012) – see discussion in Section 2.2.

[8] When competing for the revenues from an investment, if firms reach a point where there is a set of strategies with the property that no firm can benefit by changing its strategy while its opponent keeps its strategies unchanged, then that set of strategies, and the corresponding firms' payoffs, constitute a Nash equilibrium.

[9] Huisman, Kort, Pawlina, and Thijssen (2004) and Chevalier-Roignant and Trigeorgis (2011) are also literature reviews of RO models. While we focus mainly on the game theoretic aspects underlying RO models and provide full game-theoretic classification for the selected articles, Huisman et al. (2004) focus mainly on continuous-time lumpy problems, and Chevalier-Roignant and Trigeorgis (2011) are more centered on other modeling aspects of RO models such as myopic investment behavior, incremental capacity expansion and demand shocks. Smit and Trigeorgis (2006) illustrate the use of real options valuation and game theory principles.

[10] We provide in Appendix A full game-theoretic characterization for each article reviewed.

[11] In the extreme case, as soon as one firm invests, the investment becomes worthless for the other firm.