European Journal of Operational Research 237 (2014) 921–931

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Cost, revenue and profit efficiency measurement in DEA: A directional distance function approach

Biresh K. Sahoo [a,1], Mahmood Mehdiloozad [b,*], Kaoru Tone [c]

[a] Xavier Institute of Management, Bhubaneswar 751 013, India
[b] Department of Mathematics, College of Sciences, Shiraz University, Shiraz, Iran
[c] National Graduate Institute for Policy Studies, 7-22-1 Roppongi, Minato-ku, Tokyo 106-8677, Japan

ARTICLE INFO

Article history:
Received 17 January 2013
Accepted 4 February 2014
Available online 13 February 2014

Keywords:
Data envelopment analysis
Cost efficiency
Revenue efficiency
Profit efficiency
Translation invariance
Directional distance function

ABSTRACT

Estimation of efficiency of firms in a non-competitive market characterized by heterogeneous inputs and outputs along with their varying prices is questionable when factor-based technology sets are used in data envelopment analysis (DEA). In this scenario, a value-based technology becomes an appropriate reference technology against which efficiency can be assessed. In this contribution, the value-based models of Tone (2002) are extended in a directional DEA set up to develop new directional cost- and revenue-based measures of efficiency, which are then decomposed into their respective directional value-based technical and allocative efficiencies. These new directional value-based measures are more general, and include the existing value-based measures as special cases. These measures satisfy several desirable properties of an ideal efficiency measure. These new measures are advantageous over the existing ones in terms of (1) their ability to satisfy the most important property of translation invariance; (2) choices over the use of suitable direction vectors in handling negative data; and (3) flexibility in providing the decision makers with the option of specifying preferable direction vectors to incorporate their preferences. Finally, under the condition of no prior unit price information, a directional value-based measure of profit inefficiency is developed for firms whose underlying objectives are profit maximization. For an illustrative empirical application, our new measures are applied to a real-life data set of 50 US banks to draw inferences about the production correspondence of banking industry.

Introduction

Since its inception by Charnes, Cooper, and Rhodes (1978), data envelopment analysis (DEA) has been gaining increasing popularity in the literature as a convenient tool for estimating the efficiencies of firms characterized by multi-input–multi-output production technologies. The non-parametric methodology of DEA has been used for measuring and analyzing a number of efficiency concepts, including cost efficiency (CE) and revenue efficiency (RE).

One of the most important aspects in applied production analysis of firms is the measurement of their cost and revenue efficiencies (Farrell, 1957), on which we concentrate within the framework of the directional distance function (DDF) by Chambers, Chung, and Färe (1996, 1998). For the first time, Färe, Grosskopf, and Lovell (1985) developed procedures for the empirical implementations of the CE and RE measures in DEA. Since then, the aspect of measuring cost and revenue efficiencies has been explored in many studies. See, e.g., Ray and Kim (1995), Cooper, Thompson, and Thrall (1996), Schaffnit, Rosen, and Paradi (1997), Sueyoshi (1997), Puig-Junoy (2000), Kuosmanen and Post (2001), Kuosmanen and Post (2003), Tone (2002), Tone and Sahoo (2005, 2006), Maniadakis and Thanassoulis (2004), Sengupta and Sahoo (2006), Jahanshahloo, Soleimani-Damaneh, and Mostafaee (2008), Mostafaee and Saljooghi (2010), Sahoo, Kerstens, and Tone (2012), among others.

Both the CE- and RE-based DEA models developed by Färe et al. (1985) require not only input and output quantity data but also their prices at each firm. These models can be of limited use in actual applications when market imperfections exist (Camanho & Dyson, 2008; Park & Cho, 2011; Sahoo & Tone, 2013). This is because these models are based on a number of simplifying assumptions. First, factor inputs are homogeneous across firms; their prices are exogenously given, and are measured and known with full certainty. In real-life applications, however, when production

* Corresponding author. Tel.: +98 9127431689.
E-mail addresses: biresh@ximb.ac.in (B.K. Sahoo), m.mehdiloozad@gmail.com (M. Mehdiloozad), tone@grips.ac.jp (K. Tone).
[1] Tel.: +91 674 6647735; fax: +91 674 2300995.

http://dx.doi.org/10.1016/j.ejor.2014.02.017
0377-2217/

is expanded, firms experience *changes* in the organization of their processes or in the characteristics of their inputs that are economically more attractive than the replicated alternatives of those already in use. Therefore, the techniques and inputs used at higher scale are very different from those used at lower scale. Hence, factor inputs are thus heterogeneous, and as a result, their prices may vary across firms. Since inputs vary in their quality, the construction of *factor-based* technology set in DEA becomes problematic.

Further, input prices are not exogenous, but they vary in accordance with the actions by firms (Chamberlin, 1933; Engel & Rogers, 1996; Robinson, 1933). Also, firms often face *ex ante* price uncertainty while making their production decisions (McCall, 1967; Sandmo, 1971; Camanho & Dyson, 2005). While costs and revenues are all well measured, physical input and output quantities and their prices are often not. Economic theory suggests that firms enjoying some degree of monopoly power should charge different prices if there is heterogeneity in the productivity of their inputs. This is empirically valid since most firms are observed facing an upward-sloping supply curve in their purchase decisions. This observation also suggests that the assumption of facing common unit prices by firms, i.e., the law of one price, which has long been maintained as a necessary and sufficient condition for Pareto efficiency in competitive markets (Kuosmanen, Cherchye, & Sipilainen, 2006), is not at all justified in revealing the proper CE behavior of firms.

Second, the CE measure by Färe et al. (1985) can be of limited value in actual applications even when (physical) inputs are homogeneous. This is because, as pointed out by Camanho and Dyson (2008), the CE measure reflects only input inefficiencies (technical inefficiency and/or allocative inefficiency) but not market (price) inefficiencies (deviation from fully competitive setting leading to price differences between firms). Therefore, as a remedy, they suggested a comprehensive CE measure that accounts for both inputs and market inefficiencies.

Third, in many real-life applications the price data on inputs and outputs are synthetically constructed, and hence, represent average, rather than marginal prices. Since managers make decisions at the margin, analysis of efficiency using average prices can distort measures of allocative efficiency (Fukuyama & Weber, 2008).

Therefore, when inputs/outputs are heterogeneous, in order to account for situations where the input/output prices vary between firms as a result of negotiations or to reflect the qualitative differences in the resources/products, the alternative CE/RE model of Tone (2002) should be followed by setting up technology in a cost-output/input-revenue space. Using the directional DEA structure, Fukuyama and Weber (2004) and Färe and Grosskopf (2006) extended this alternative value-based CE model to develop the directional input-cost distance function (DICDF), which, in turn, provides a directional measure of value-based technical inefficiency.

Using the DICDF, we develop two new directional cost- and revenue-based measures of efficiency, i.e., DCE and DRE, which all satisfy the property of translation invariance. This property is considered most important for any efficiency measure (Ali & Seiford, 1990; Cooper, Park, & Pastor, 1999; Lovell & Pastor, 1995; Pastor, 1996). Furthermore, we develop two new directional input- and output-oriented value-based measures of technical efficiency (TE). We then decompose our new DCE and DRE measures into their respective directional value-based TE and allocative efficiency (AE) components. These new DCE and DRE measures are more general, and include the Tone (2002)'s CE and RE measures as special cases. Our proposed new measures satisfy several desirable properties, such as unit invariance (Cooper et al., 1999; Lovell & Pastor, 1995) and strong monotonicity (Blackorby & Russell, 1999; Cooper et al., 1999).

Note that our value-based DCE measure is developed based on the assumption that physical outputs are homogenous, but not physical inputs. Similarly, the value-based DRE measure is developed based on the assumption that physical inputs are homogenous, but not physical outputs. However, when both physical inputs and outputs are heterogeneous, our DCE and DRE measures cannot be applied to measure the respective cost and revenue efficiencies. To deal with this situation, we develop a directional value-based measure of profit (in)efficiency that is based on a technology set comprising of all feasible input-cost (input-spending) and output-revenue (output-earnings) by observed firms. This measure will be more meaningful for a firm when its underlying behavioral objective is profit maximization.

While none of the existing CE, RE, and AE measures is translation-invariant, our proposed new measures satisfy this property that enables them to effectively deal with negative data. These new measures are flexible in the sense that they provide the decision makers with the option of specifying preferable direction vectors to incorporate their decision-making preferences. Specially, they can deal with value judgments (preference) as to which specific input-cost to reduce or which specific output-earnings to increase by a firm to improve its overall performance. Though the contribution of this paper is mainly theoretical, to demonstrate its ready applicability in empirical work, we conduct an illustrative empirical analysis based on a data set of 50 US banks.

The remainder of the paper unfolds as follows. Section 'Preliminaries' gives a brief review of methods aimed at measuring CE and RE. Section 'Our Proposed Approach' represents the main contribution of the paper, where we present our new directional CE, RE and profit (in)efficiency measures and then discuss their properties. Section 'An Empirical Illustration' demonstrates the ready applicability of our proposed measures on a real-life data set of 50 US banks for the year 1996. Finally, Section 'Concluding Remarks' concludes with remarks.

Preliminaries

Throughout this paper, we assume to deal with n observed decision making units (DMUs); each uses m inputs to produce s outputs. Let $x_j = (x_{1j}, \ldots, x_{mj})^T \in \mathbb{R}^m_{\geqslant 0}$ and $y_j = (y_{1j}, \ldots, y_{sj})^T \in \mathbb{R}^s_{\geqslant 0}$ be, respectively, the input and output vectors of DMU$_j$, $j \in J = \{1, \ldots, n\}$. Let $c_j = (c_{1j}, \ldots, c_{mj})^T \in \mathbb{R}^m_{\geqslant 0}$ and $p_j = (p_{1j}, \ldots, p_{sj})^T \in \mathbb{R}^s_{\geqslant 0}$ be, respectively, the non-negative price vectors of input and output of DMU$_j$. The superscript T stands for a vector transpose. Let the input-spending and output-earnings of DMU$_j$ be $\bar{x}$ and $\bar{y}$ respectively, where $\bar{x} = c * x$ and $\bar{y} = p * y$. Here, $*$ denotes the component-wise multiplication of vectors. We further use o as the index of DMU under evaluation.

We now define four production technologies depending upon data availability. If both physical input and output data are observed, and are homogeneous, we represent technology as

$$T_{x,y} = \left\{(x, y) \in \mathbb{R}^{m+s}_{\geqslant 0} \middle| x \in \mathbb{R}^m_{\geqslant 0} \text{ can produce } y \in \mathbb{R}^s_{\geqslant 0}\right\}. \quad (1)$$

If physical outputs are observed (and are homogeneous) but not physical inputs, then we can represent the technology by considering all feasible input-spending and physical output vectors as

$$T_{\bar{x},y} = \left\{(\bar{x}, y) \in \mathbb{R}^{m+s}_{\geqslant 0} \middle| \bar{x} \in \mathbb{R}^m_{\geqslant 0} \text{ can produce } y \in \mathbb{R}^s_{\geqslant 0}\right\}. \quad (2)$$

If physical inputs are observed (and are homogeneous) but not physical outputs, then we can represent the technology by considering all feasible physical input and output-earnings vectors as

$$T_{x,\bar{y}} = \left\{(x, \bar{y}) \in \mathbb{R}^{m+s}_{\geqslant 0} \middle| x \in \mathbb{R}^m_{\geqslant 0} \text{ can produce } \bar{y} \in \mathbb{R}^s_{\geqslant 0}\right\}. \quad (3)$$