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Short Communication

A new distance measure including the weak preference relation: Application to the multiple criteria aggregation procedure for mixed evaluations

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ABSTRACT

We introduce a new distance measure between two preorders that captures indifference, strict preference, weak preference and incomparability relations. This measure is the first to capture weak preference relations. We illustrate how this distance measure affords decision makers greater modeling power to capture their preferences, or uncertainty and ambiguity around them, by using our proposed distance measure in a multiple criteria aggregation procedure for mixed evaluations.

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1. Introduction

An abundant literature exists addressing the problem of appropriate distance measures between two preorders. The context of much of this literature is group decision making where individual preorders have to be reconciled into a collective or compromised preorder. Kemeny and Snell (1962) were the first to use a distance-based model for this purpose, presenting a set of conditions that a distance measure must satisfy. Many authors, such as Cook and Seiford (1978), Cook and Kress (1985) amongst others, have proposed similar conditions to those of Kemeny and Snell (1962) to construct different distance measures between two total preorders (including the indifference and the preference relations). However, few studies have examined the case of partial preorders (including the incomparability relation). Bogart (1975) generalized the model of Kemeny and Snell (1962) to accommodate partial preorders but excluded the indifference relation. Cook, Kress, and Seiford (1986a, 1986b) included the indifference relation presenting a set of axioms showing the existence and uniqueness of a distance measure between two individual preorders. Yet they did not address the aggregation problem for determining a collective preorder. In a multicriteria decision-aiding context, the aggregation problem with partial preorders was addressed by constructing convex cones to partially order the set of alternatives (Dehnokhalaji, Korhonen, Köksalan, Nasrabadi, & Wallenius, 2011). In a multicriteria analysis on water supply systems, Roy and Slowinski (1993) were the first to introduce the idea of a distance measure between pairs of binary relations. Their approach was adapted by Ben Khélifa and Martel (2001) to tackle the aggregation problem and an algorithm proposed to determine a total collective preorder from partial individual preorders. Jabeur, Martel, and Ben Khélifa (2004) presented a new distance measure considered to improve these previous models. Indeed, they proposed a minimal set of conditions to construct a metric. Recently, they used the same set of conditions to assign new values to their distance measure (Jabeur & Martel, 2010). Other approaches to measure the distance between preference relations have been proposed but they do not include the incomparability relation (Meskanen & Nurmi, 2006; Nitzan, 1981).

We introduce a new distance measure between preorders, including the weak preference relation, extending the work of Jabeur et al. (2004). The weak preference relation (Q) is an intermediate relation between preference and indifference first introduced by Roy (1978) with the ELECTRE III method. It is meant to capture situations where distinguishing between preference and indifference is problematic because of ambiguity and/or uncertainty.

Section 2 presents the new distance measure D including the weak preference relation. It is used in the Multiple Criteria Aggregation Procedure (MCAP) for mixed evaluations (Ben Amor, Jabeur, & Martel, 2007) to aggregate n unicriterion (local) preorders into a

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multicriterion (global) preorder. The MCAP is extended here to include the weak preference relation to afford the decision maker greater modeling power to elucidate preferences. Several changes required for this purpose and an illustrative numerical example, are presented in Section 3. Conclusions and future work follow in Section 4.

2. Extended distance measure *D*

Let A be a finite set of objects (alternatives, options or actions in this context) and (a_i, a_k) an ordered pair of objects belonging to A. In order to set a preference between a_i and a_j, four binary relations are considered: the strict preference (P), the weak preference (Q), the indifference (I) and the incomparability (?). For the sake of convenience, we use $a_i\ P^{-1}\ a_k$ for $a_k\ P\ a_i$ and $a_i\ Q^{-1}\ a_k$ for $a_k\ Q\ a_i$, P^{-1} and Q^{-1} stand for the inverse strict preference and the inverse weak preference respectively. Based on Jabeur et al. (2004), we propose a set of seven "logical" conditions (an axiomatic) allowing to compare the distance between each pair of binary relations $\{P, P^{-1}, Q, Q^{-1}, I, ?\}$. If distance D satisfies these conditions, it can be easily proven that D is a metric, i.e., it verifies the non-negativity, the symmetry and the triangular inequality properties.

Condition 1: (C1).

$$D(P, ?) = D(P^{-1}, ?) \quad \text{and} \quad D(P, I) = D(P^{-1}, I) \tag{1}$$

$$D(Q, ?) = D(Q^{-1}, ?) \quad \text{and} \quad D(Q, I) = D(Q^{-1}, I) \tag{2}$$

$$D(P, Q^{-1}) = D(Q, P^{-1}) \tag{3}$$

This condition is natural since the strict preference P and the inverse strict preference P^{-1}, as the weak preference Q and the inverse weak preference Q^{-1} are symmetrical to each other ($a_i P a_k \iff a_k P^{-1} a_i$ and $a_i Q a_k \iff a_k Q^{-1} a_i$).

Condition 2: (C2).

$$D(P, P^{-1}) = Max\{D(O, U)/O, U \in \{P, P^{-1}, Q, Q^{-1}, I, ?\}\} \tag{4}$$

Eq. (4) indicates that the strict preference and the inverse strict preference relations are the most discordant relations.

Condition 3: (C3).

$$D(O, U) > 0 \text{ if } O \neq U \text{ and } D(O, U) = 0 \text{ if } O \equiv U \text{ when } O, U \in \{P, P^{-1}, Q, Q^{-1}, I, ?\} \tag{5}$$

This condition states that the minimum distance between two distinct relations is positive. It is null in the opposite case.

Condition 4: (C4).

$$D(P, ?) = D(Q, ?) = D(I, ?) \tag{6}$$

Following the interpretation of several authors including Roy and Bouyssou (1993) and Schärlig (1996), the incomparability is seen as the affirmation of the incapacity to establish the relation type: there is no indifference, no weak preference and no strict preference between the two alternatives under consideration. Thus, in the absence of any additional information, the incomparability relation should be considered as equidistant from the other preference relations, in this case $\{P, P^{-1}, Q, Q^{-1}, I\}$. This consideration relies on the insufficient reason principle of Laplace.[1] Jabeur et al. (2004) also followed this line of argument. They considered that the incomparability relation is equidistant from the indifference, the strict preference and the inverse strict preference relations. Roy and Slowinski (1993) as well as Ben Khélifa and Martel (2001) stipulated that the passage from indifference to incomparability is less demanding than the passage from preference to incomparability, i.e., $D(P, ?) \geqslant D(I, ?)$.

Condition 5: (C5).

$$D(Q, P) = D(Q, I) \tag{7}$$

This condition states that the weak preference is at an equal distance from the indifference and the strict preference. One can easily state that the weak preference relation lies between the indifference and the strict preference relation. The hypothesis about equidistance is reasonable if one thinks of the weak preference relation as a state of mind expressing hesitation between indifference and strict preference (Roy, 1978). No reason exists to place the weak preference relation in a closer position to one or the other of the two preference relations considered here. Consequently, Laplace's principle of insufficient reason can be invoked to justify equidistance.

Condition 6: (C6).

$$D(P, ?) \geqslant D(P, I) \tag{8}$$

$$D(?, I) \geqslant D(P, I) \tag{9}$$

$$D(P, I) \geqslant D(Q, I) \tag{10}$$

$$D(Q^{-1}, P) \geqslant D(Q, P) \tag{11}$$

$$D(P, Q^{-1}) \geqslant D(P, I) \tag{12}$$

$$D(P, Q^{-1}) \geqslant D(Q, Q^{-1}) \tag{13}$$

$$D(I, ?) \geqslant D(Q, Q^{-1}) \tag{14}$$

Inequality (8) was justified by Jabeur et al. (2004). Eq. (9) follows considering condition 4, while inequalities (10)–(13) are consistent conditions considering the preference relations at hand. For instance, the passage from strict preference to indifference is more demanding than the passage from weak preference to indifference. For the sake of consistency and without loss of generality, we assume that Eq. (14) holds with strict inequality.

Graphically, the previous conditions are illustrated in Fig. 1 where:

- $D(P,?) = D(Q,?) = D(I,?) = D(?,Q^{-1}) = D(?,P^{-1}) = x$ (by C4).
- $D(P,Q) = D(Q,I) = D(I,Q^{-1}) = D(Q^{-1},P^{-1}) = a$ (by C1 and C5).
- $D(P,I) = D(P^{-1},I) = b$ (by C1).

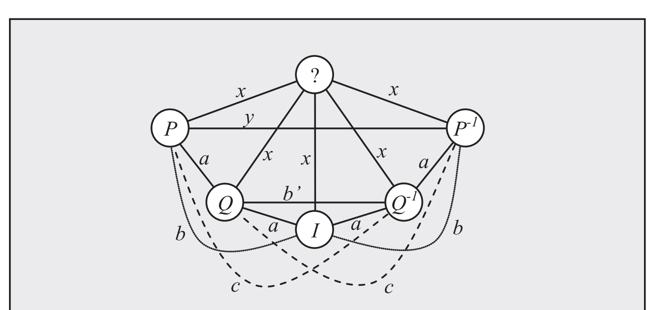

Fig. 1. Graphic representation of conditions C1–C7.

[1] The Principle of Insufficient Reason states that if there is no reason to believe that out of a set of possible, mutually exclusive events no one event is more likely to occur than any other, then one should assume that all events have the same objective probability (Laplace, 1814).

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