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Discrete Optimization

The mixed capacitated general routing problem under uncertainty

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ABSTRACT

We study the General Routing Problem defined on a mixed graph and with stochastic demands. The problem under investigation is aimed at finding the minimum cost set of routes to satisfy a set of clients whose demand is not deterministically known. Since each vehicle has a limited capacity, the demand uncertainty occurring at some clients affects the satisfaction of the capacity constraints, that, hence, become stochastic. The contribution of this paper is twofold: firstly we present a chance-constrained integer programming formulation of the problem for which a deterministic equivalent is derived. The introduction of uncertainty into the problem poses severe computational challenges addressed by the design of a branch-and-cut algorithm, for the exact solution of limited size instances, and of a heuristic solution approach exploring promising parts of the search space. The effectiveness of the solution approaches is shown on a probabilistically constrained version of the benchmark instances proposed in the literature for the mixed capacitated general routing problem.

1. Introduction

An important operative issue in the context of the distributive logistics consists in planning the delivery routes performed by a fleet of vehicles to satisfy the requests of a set of elements of a network, namely required vertices, edges and arcs. In mathematical terms, the problem is modeled as *Mixed Capacitated General Routing Problem* (*MCGRP*): it basically consists in finding a set of routes on a mixed graph, beginning and ending at the same vertex (depot), with minimum total cost, satisfying demands located at links and vertices and with a capacity restriction on the demand satisfied by each route. The *MCGRP* generalizes many vehicle routing problems that have been widely studied in the last forty years and for which hundreds of papers have been written, either to give exact or heuristic procedures for their resolution or to provide lower bounds. Despite the practical importance of the mixed general routing problem, relatively few studies have been published on it. Most works deal with the uncapacitated case. Corberán, Letchford, and Sanchis (2001, 2003, 2005) studied the feasible polyhedron starting from an integer programming formulation solved through an efficient cutting-plane algorithm. Blais and Laporte (2003) proposed a different approach based on the transformation of the original problem into an equivalent Traveling Salesman Problem or Rural Postman Problem which are solved in turn through available exact algorithms.

With respect to the capacitated case, Bosco, Laganà, Musmanno, and Vocaturo (2013) proposed a novel integer programming formulation and a branch-and-cut algorithm (*B&C*) where surrogate inequalities, introduced for the Capacitated Arc Routing Problem, are extended to the *MCGRP* polyhedron.

The aim of this paper is the introduction of the uncertainty issue in this latter and more involved case, where each vehicle has a limited capacity. In effect, most of the real-world applications modeled as *MCGRP* are characterized by some uncertainty which affects the customers' demand. For example, the operational plan of pickup routes in solid waste collection systems implies modeling the service by the means of required arcs or edges whenever the collection points are distributed along the streets, while some vertices are required if the collection is concentrated around specific points (e.g., hospitals, schools, and supermarkets). For generality, we shall also assume that the requests of random elements might be correlated to faithfully represent real situations. For example, in the garbage collection, the geographical nearness of some customers within the same regional district or along the same street suggests to consider a statistical correlation among their garbage productions.

Following these considerations, we bring the stochasticity into the *MCGRP* by adopting the paradigm of the probabilistic constraints defined within the general Stochastic Programming (SP) framework (Birge & Louveaux, 1997). This modeling paradigm is appropriate in many situations, where an operational plan is

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periodically updated over a long planning horizon, and hence, becomes crucial to design a set of a priori routes that will cover the uncertain requests with a high reliability level. In particular, we formally introduce a stochastic formulation of the *MCGRP* where the stochastic capacity constraints are re-formulated in terms of probabilistic constraints. The explicit inclusion of the uncertainty within an already proved NP-hard problem, poses additional challenges, calling for the design of tailored solution approaches. This represents the second core contribution of the present paper. We develop a branch-and cut (*B*&*C*) algorithm for solving small instances and we design a large neighborhood search heuristic for the solution of instances of larger size where the *B*&*C* algorithm is used, in turn, to perform an exact local search on a portion of the overall feasible region.

To put our contribution in the right perspective, we should precise that the adoption of the SP framework to model routing problems under uncertainty is not completely new. For an extensive survey, the readers are referred to Dror and Trudeau (1986) and Gendreau, Laporte, and Seguin (1996).

Within this stream, most of the contributions rely on the two-stage paradigm and different recourse policies have been proposed in the literature. Bertsimas (1992) and Bertsimas and Simchi-Levi (1994) focused their researches on simple recourse policies that are separable by vehicle. A different policy has been presented by Ak and Erera (2007), that proposed a two-vehicle sharing recourse policy. During the last decades various heuristic and exact optimization approaches have been proposed and analyzed for constructing a set of tours minimizing expected costs given this recourse policy. Gendreau, Laporte, and Seguin (1995) proposed an exact solution for an a priori optimization model based on an integer L-shaped method. Laporte, Louveaux, and van Hamme (2002) presented an improved method where strong lower bounds at the root node contribute significantly to speed up the solution times. Gendreau, Laporte, and Seguin (1996) applied local search concepts embedded into a tabu search scheme to solve the a priori model presented in Gendreau et al. (1995). More recently, Laporte, Musmanno, and Vocaturo (2010) studied the capacitated arc-routing problem with stochastic demands in the context of garbage collection and proposed an adaptive large neighborhood search heuristic.

Scant attention has been devoted to the formulation of routing problems with probabilistic constraints. Stewart and Golden (1983) presented a model able to find minimum cost routes with a threshold constraint on the probability of a route failure, whereas Laporte, Louveaux, and Mercure (1989) proposed a chance-constrained model for location-routing problems. A chance constrained version of the vehicle routing problem, solved to optimality by algorithms similar to those developed for the deterministic case, has been presented in Dror, Laporte, and Louveaux (1993).

Besides the stochastic programming approach, the robust optimization framework has been adopted to deal with routing problems involving uncertain parameters where the probability distributions are not known. Amongst the recent contributions, we cite Sungur, Ordóñez, and Dessouky (2008), who analyze the case of uncertain customer demands and travel times. The goal is to determine vehicle routes which satisfy the capacity constraints and the specified time windows if all the uncertain parameters attain the worst case realizations simultaneously. The problem can be simplified to a deterministic model, which is attractive from a computational standpoint. Gounaris, Wiesemann, and Floudas (2013) (see also the references therein) investigate the case of capacitated vehicle routing problem. Robust optimization counterparts of several deterministic formulations of the problem are derived and numerically compared. Robust rounded capacity inequalities are developed, which can be separated efficiently for two broad classes of demand supports. Finally, the authors analyze the relation between the robust models and the chance constrained counterparts. Lee, Lee, and Park (2004) considered two types of uncertainty sets for the possible realizations of travel times and demands. The authors propose a column generation algorithm which encapsulates the robustness in the pricing problem cast as a robust version of the shortest path with resource constraints.

In this paper we study the Mixed Capacitated General Routing Problem with Probabilistic Constraints *MCGRPPC*. In Section 2, we introduce the problem and we provide a chance-constrained integer linear programming formulation for the *MCGRPPC*. In Section 3 we define the *B*&*C* algorithm for solving small instances of the *MCGRPPC*. In Section 4 we present a tailored heuristic search to solve larger *MCGRPPC* instances. In Section 5, we present the results of our computational study. Finally, in Section 6, we give our conclusions and discuss future perspectives in this area.

2. Problem description

The *MCGRPPC* is defined over a mixed graph $G = (V, A, E)$, where $V = \{1, \ldots, n\}$ represents the set of vertices, where vertex 1 represents the depot, and $A = \{(i,j) \subseteq V \times V\}$ is the set of arcs, whereas $E = \{(i,j) \subseteq V \times V : i < j\}$ is the set of edges.

In the following, we shall denote by $L = A \cup E$, the set of *links* and we shall indicate by c_{ij} a non-negative cost coefficient associated with each link (i,j). We assume that the service activity may occur at some vertices $V_R \subseteq V$, named *required vertices*, *arcs* $A_R \subseteq A$ and/or *edges* $E_R \subseteq E$, named *required arcs* and *required edges*, respectively. Thus, $L_R = A_R \cup E_R$ denotes the set of *required links* of G and all the required vertices and links will be referred to as required *elements* and indicated by R.

For each subset $S \subset V$ of vertices, or its complementary set $\overline{S}(\overline{S} = V \setminus S)$, we define the following sets:

(a) $\delta^+(S) = \{(i,j) \in A : i \in S \wedge j \in \overline{S}\}$,
(b) $\delta^-(S) = \{(i,j) \in A : i \in \overline{S} \wedge j \in S\}$,
(c) $\delta^+_{A_R}(S) = \{(i,j) \in A_R : i \in S \wedge j \in \overline{S}\}$,
(d) $\delta^-_{A_R}(S) = \{(i,j) \in A_R : i \in \overline{S} \wedge j \in S\}$,
(e) $\delta(S) = \{(i,j) \in E : i \in S \wedge j \in \overline{S},\ \texttt{or}\ i \in \overline{S} \wedge j \in S\}$,
(f) $\delta_{E_R}(S) = \{(i,j) \in E_R : i \in S \wedge j \in \overline{S},\ \texttt{or}\ i \in \overline{S} \wedge j \in S\}$,
(g) $\delta_L(S) = \delta^+(S) \cup \delta^-(S) \cup \delta(S)$,
(h) $\delta_{L_R}(S) = \delta^+_{A_R}(S) \cup \delta^-_{A_R}(S) \cup \delta_{E_R}(S)$,
(i) $S_R = S \cap V_R$,
(j) $A_R(S) = \{(i,j) \in A_R : i \in S \wedge j \in S\}$,
(k) $E_R(S) = \{(i,j) \in E_R : i \in S \wedge j \in S\}$,
(l) $R(S) = A_R(S) \cup E_R(S) \cup S_R$.

The previous notation remains valid as long as S is replaced by v, and $\overline{S}$ by $\overline{v}$, or $V \setminus \{v\}$. We denote by G^R the graph induced on G by all the required links and vertices. Generally, this graph is non-connected. The vertex sets corresponding to connected components of G^R are called R-sets. The subgraphs of G induced by the R-sets define the so-called R-connected components of G. An isolated required vertex represents itself an R-connected component of G.

In real settings, the service demand associated with all but a subset of required elements is seldom, if ever, known at the time routes have to be designed. Thus, with the aim of more realistically modeling general routing problems, one should deal with the stochastic nature of the input parameters. In the following, we shall assume that the set of required elements is partitioned into two subsets R_C and R_U to differentiate between elements with known and uncertain demands, respectively. Following the stochastic programming modeling framework, we shall assume that the uncertain demands are represented in terms of random variables