European Journal of Operational Research 240 (2015) 488–492

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Decision Support

Cost efficiency in data envelopment analysis under the law of one price

Lei Fang [a,*], Hecheng Li [b]

[a] Business School, Nankai University, People's Republic of China
[b] Department of Mathematics, Qinghai Normal University, People's Republic of China

ARTICLE INFO

Article history:
Received 11 October 2013
Accepted 11 July 2014
Available online 18 July 2014

Keywords:
Data envelopment analysis
Law of one price
Cost efficiency

ABSTRACT

To impose the law of one price (LoOP) restrictions, which state that all firms face the same input prices, Kuosmanen, Cherchye, and Sipiläinen (2006) developed the top-down and bottom-up approaches to maximizing the industry-level cost efficiency. However, the optimal input shadow prices generated by the above approaches need not be unique, which influences the distribution of the efficiency indices at the individual firm level. To solve this problem, in this paper, we developed a pair of two-level mathematical programming models to calculate the upper and lower bounds of cost efficiency for each firm in the case of non-unique LoOP prices while keeping the industry cost efficiency optimal. Furthermore, a base-enumerating algorithm is proposed to solve the lower bound models of the cost efficiency measure, which are bi-level linear programs and NP-hard problems. Lastly, a numerical example is used to demonstrate the proposed approach.

1. Introduction

The concept of cost efficiency (CE), which evaluates the ability of a DMU to produce the current outputs at a minimal cost given the input price at each DMU, originates from Farrell (1957). Based on this concept, the cost efficiency is conventionally defined as the ratio of the minimum cost to the actual observed cost (see Thompson, Brinkmann, Dharmapala, Gonzalez-Lima, and Thrall (1997), Schaffnit et al. (1997), Taylor, Thompson, Thrall, and Dharmapala (1997), Camanho and Dyson (2005), Cherchye and Vanden Abeele (2005), and Jahanshahloo, Soleimani-damaneh, and Mostafaee (2008)). CE models require that all of the input prices be fixed and known exactly at each DMU. However, in their actual application, exact knowledge of the prices is difficult, and prices may be subject to variations in the short term, as noted by Cooper, Thompson, and Thrall (1996). To address the uncertain price data, some researchers have developed models to obtain the upper and lower bounds of the CE (Camanho & Dyson, 2005; Fang & Li, 2012, 2013; Mostafaee & Saljooghi, 2010).

Note that the input prices in the above papers are allowed to differ across firms. However, much of the price variation across firms is at odds with the common perception of price information in competitive markets (Kuosmanen, Cherchye, & Sipiläinen, 2006). In a recent study, Kuosmanen et al. (2006) explored the productive efficiency analysis under the law of one price (LoOP), which refers to the same prices of the inputs for all of the firms under market equilibrium. Utilizing the relationship between industry-level and firm cost efficiency measures, they developed the top-down and the bottom-up approaches to maximize the industry-level cost efficiency with respect to the LoOP condition under incomplete price information. However, the optimal shadow prices generated by the above approaches need not be unique, which may affect the distribution of the efficiency indices at the firm level (Fang, 2013; Kuosmanen, Kortelainen, Sipiläinen, & Cherchye, 2010). These researchers noted that an interesting avenue for follow-up research is to develop an efficient algorithm to calculate the upper and lower bounds of cost efficiency for each firm in the case of non-unique LoOP prices. However, calculating these bounds is extremely complicated because changing the input prices influences not only the actual cost but also the minimum cost for producing the given output (Kuosmanen et al., 2006).

To solve the above problems, in this paper, we developed a pair of two-level mathematical programming models to calculate the upper and lower bounds of cost efficiency for each firm in the case of non-unique LoOP prices while keeping the industry cost efficiency at its optimal value.

The paper is organized as follows: Section 2 presents a pair of DEA models to calculate the upper and lower CE measures for each firm under the framework of the top-down approach in the case of non-unique LoOP prices. A base-enumerating algorithm based on the optimality conditions of linear programming is proposed to solve the lower CE models in Section 3. Section 4 illustrates the proposed approaches with a numerical example. Section 5 concludes the paper.

* Corresponding author. Tel./fax: +86 2223505660.
E-mail address: fanglei@nankai.edu.cn (L. Fang).

http://dx.doi.org/10.1016/j.ejor.2014.07.017
0377-2217/

2. The proposed approach

Suppose an industry consists of n decision-making units. Each DMU j ($j = 1, \ldots, n$) utilizes the inputs $x_j = (x_{j1}, \ldots, x_{jR})$ to produce the outputs $y_j = (y_{j1}, \ldots, y_{jS})$.

Assume that the technology exhibits constant returns-to-scale and that all DMUs operate under the same technology. According to the relationship between firm-level efficiency indices and industry-level efficiency reported by Li and Ng (1995), Ylvinger (2000) and Fare and Zelenyuk (2003), the industry technology equals the individual firm technology. Kuosmanen et al. (2006) developed the following top-down approach to estimate the industry cost efficiency:

$$\mathrm{ITCE}^{\mathrm{FDH}}\ (\chi, \eta, W) = \max\ c \tag{1}$$

$$\text{s.t. } c \leqslant \sum_{s=1}^{S} \eta_s p_{js} \quad j = 1, \ldots, n$$

$$\sum_{s=1}^{S} y_{js} p_{js} - \sum_{r=1}^{R} x_{jr} w_r \leqslant 0 \quad j = 1, \ldots, n$$

$$\sum_{r=1}^{R} \chi_r w_r = 1$$

$$\mathbf{w} \in W$$

where $(\chi, \eta) = \left(\sum_{j=1}^{n} x_j, \sum_{j=1}^{n} y_j\right)$ is the aggregated industry input–output combination. p_{js} represents the shadow price of output s for DMU j; $W \equiv \left\{ w \in R_{+}^{R} | \mathbf{A}w' \geqslant \mathbf{b}| \right\}$ represents the polyhedral convex set for the input price domain; $\mathbf{A}$ is an $l \times R$ matrix and $\mathbf{b}$ an l-dimensional column vector. The third constraint guarantees the input prices are consistent with LoOP. Let c^* be the optimal value to the model (1).

Assuming w_r^* ($r = 1, \ldots, R$) are the optimal input shadow prices that maximize the industry cost efficiency of model (1), we can use the optimal input shadow prices, which are assumed to be unique, to estimate the firm-level efficiency analysis. Unfortunately, the optimal input shadow prices are not always unique, making the cost efficiency levels of individual firms uncertain (Mostafaee & Saljooghi, 2010).

In the following, we shall develop optimistic and pessimistic DEA models to obtain the upper and lower bounds of cost efficiency, respectively, for each firm in the case of non-unique LoOP prices in model (1).

To obtain the upper bound of cost efficiency for DMU j, we can propose the optimistic CE model in the following mathematical formulation:

$$\mathrm{UTCE}^{\mathrm{FDH}}(x_j, y_j, W) = \max\ c_j \tag{2}$$

$$\text{s.t. } c^* \sum_{r=1}^{R} \chi_r w_r \leqslant \sum_{s=1}^{S} \eta_s p_{ms} \quad m = 1, \ldots, n \tag{2.1}$$

$$c_j \leqslant \sum_{s=1}^{S} y_{js} p_{ms} \quad m = 1, \ldots, n \tag{2.2}$$

$$\sum_{s=1}^{S} y_{ms} p_{ms} - \sum_{r=1}^{R} x_{mr} w_r \leqslant 0 \quad m = 1, \ldots, n \tag{2.3}$$

$$\sum_{r=1}^{R} x_{jr} w_r = 1 \tag{2.4}$$

$$\mathbf{p}_m \in R_{+}^{S} \quad m = 1, \ldots, n$$
$$\mathbf{w} \in W$$

where $\mathbf{p}_m$ represents the output shadow price vector for DMU m. The above optimistic model takes the most optimistic strategy, which selects the most favorable input prices among the multiple input shadow prices of model (1) to evaluate DMU j. We can thus refer to it as an upper bound for the cost efficiency of the evaluated DMU j.

Let w_r^* $(r = 1, \ldots, R)$ and $p_{ms}^* (m = 1, \ldots, n,\ s = 1, \ldots, S)$ be the optimal solution to model (2).

The following **theorem** shows that the above optimal shadow prices keep the industry cost efficiency at the optimal value c^* of model (1).

Theorem 1. *If w_r^* $(r = 1, \ldots, R)$ and p_{ms}^* $(m = 1, \ldots, n,\ s = 1, \ldots, S)$ solve model (2), then the industry cost efficiency corresponding to the optimal shadow prices equals c^*.*

Proof. Let w_r^* $(r = 1, \ldots, R)$ and p_{ms}^* $(m = 1, \ldots, n, s = 1, \ldots, S)$ be the optimal solution to model (2). Assume that $\sum_{r=1}^{R} \chi_r w_r^* = \rho$. According to constraints (2.1) and (2.3), we have

$$c^* \rho \leqslant \sum_{s=1}^{S} \eta_s p_{ms}^* \quad m = 1, \ldots, n \tag{2.5}$$

$$\sum_{s=1}^{S} y_{ms} p_{ms}^* - \sum_{r=1}^{R} x_{mr} w_r^* \leqslant 0 \quad m = 1, \ldots, n \tag{2.6}$$

Now, we define $w_r^{**} = \frac{1}{\rho} w_r^*$ and $p_{ms}^{**} = \frac{1}{\rho} p_{ms}^*$. Thus, we have $\sum_{r=1}^{R} \chi_r w_r^{**} = 1$. In addition, dividing both sides of the constraints of (2.4) and (2.5) by ρ, we have

$$c^* \leqslant \sum_{s=1}^{S} \eta_s p_{ms}^{**} \quad m = 1, \ldots, n$$

$$\sum_{s=1}^{S} y_{ms} p_{ms}^{**} - \sum_{r=1}^{R} x_{mr} w_r^{**} \leqslant 0 \quad m = 1, \ldots, n$$

Thus, $\left(w_r^{**},\ r = 1, \ldots, R;\ p_{ms}^{**},\ m = 1, \ldots, n,\ s = 1, \ldots, S\right)$ is a feasible solution to model (1). Moreover, the objective function of model (1) corresponding to this solution equals c^*.

This completes the proof. □

If we take the least favorable strategy in choosing the input prices among the multiple input shadow prices of model (1), then a Pessimistic CE model is generated. In this problem, once w_r $(r = 1, \ldots, R)$ and p_{ms} $(m = 1, \ldots, n, s = 1, \ldots, S)$ are fixed, one can obtain a maximum c_j; when w_r and p_{ms} are fixed to other values satisfying the constraints, another maximal c_j is obtained. Our purpose is to find the minimal maximal value of c_j. Thus, the pessimistic CE model is a min–max problem, which is in essence a bi-level programming problem. To obtain the lower bound of the cost efficiency for DMU j, the following bi-level programming model is presented:

$$\mathrm{LTCE}^{\mathrm{FDH}}\ (x_j, y_j, W) = \min c_j \tag{3}$$

$$\text{s.t. } c^* \sum_{r=1}^{R} \chi_r w_r \leqslant \sum_{s=1}^{S} \eta_s p_{ms} \quad m = 1, \ldots, n$$

$$\max\ c_j$$

$$\text{s.t. } c_j \leqslant \sum_{s=1}^{S} y_{js} p_{ms} \quad m = 1, \ldots, n$$

$$\sum_{s=1}^{S} y_{ms} p_{ms} - \sum_{r=1}^{R} x_{mr} w_r \leqslant 0 \quad m = 1, \ldots, n$$

$$\sum_{r=1}^{R} x_{jr} w_r = 1$$

$$\mathbf{p}_m \in R_{+}^{S} \quad m = 1, \ldots, n$$
$$\mathbf{w} \in W$$