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Decision Support

Robust equilibria in location games

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ABSTRACT

In the framework of spatial competition, two or more players strategically choose a location in order to attract consumers. It is assumed standardly that consumers with the same favorite location fully agree on the ranking of all possible locations. To investigate the necessity of this questionable and restrictive assumption, we model heterogeneity in consumers' distance perceptions by individual edge lengths of a given graph. A profile of location choices is called a "robust equilibrium" if it is a Nash equilibrium in several games which differ only by the consumers' perceptions of distances. For a finite number of players and any distribution of consumers, we provide a complete characterization of robust equilibria and derive structural conditions for their existence. Furthermore, we discuss whether the classical observations of minimal differentiation and inefficiency are robust phenomena. Thereby, we find strong support for an old conjecture that in equilibrium firms form local clusters.

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1. Introduction

In his classic example, Harold Hotelling illustrates competition in a heterogeneous market by two firms that consider where to place their shop on a main street (Hotelling, 1929). Ever since, this model of spatial competition has inspired a tremendous amount of research in various disciplines. Starting with Downs (1957), it is used to analyze the positioning of political candidates competing for voters (e.g., Mueller, 2003; Roemer, 2001) and to analyze the positioning of products in order to attract consumers (e.g., Carpenter, 1989; Salop, 1979). In the year 2013 alone, Hotelling has been cited more than 450 and Downs even more than 1100 times.[1] Moreover, the model implication of *minimal differentiation* is known far beyond scholarly circles. In this paper, we want to challenge a fundamental aspect of the Hotelling–Downs approach.

Throughout the literature (of spatial competition), it has been virtually always assumed that consumers or voters who prefer the same position fully agree upon the ranking of the other alternatives, i.e., they have identical preferences or utility functions. This very strong homogeneity requirement can be considered as driven by the assumption that all consumers/voters use the same distance measure since in the standard Hotelling–Downs set-up (dis)utility is represented by the distance between positions. In particular, if two people prefer the same option, in any spatial representation with homogeneous distances they necessarily rank all the other alternatives in the same order. This is hard to justify when we think of voters of the same political party who disagree about the second-best party, or of consumers with the same favorite brand but disagreement about the ordering of two other brands. And even in the case of geographic location choices the requirement appears to be challengeable if the distances represent travel time, for instance.[2] As a matter of fact, these simple cases already exceed the scope of almost any model of locational competition.

Consider, for example, a poll on a group of voters about their favorite tax rate. The answers can be displayed as locations on a line. Location games that capture this application consider classically two political candidates who strategically choose a tax rate which they propose to the voters. Thereby it is standardly assumed that (a) each voter casts his vote for the candidate that is closest to him and (b) all voters asses the distances between the candidates homogeneously. In combination these two assumptions are not at all innocuous. As indicated above, they hide the homogeneity requirement that all voters who consider a tax rate of 10%, for instance, as their favorite alternative, are supposed to rank any two tax rates, like 2% and 20%, for example, in exactly the same

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[1] Google Scholar, February 10, 2014.

[2] Indeed, it is possible that two individuals differ in their speed of walking uphill such that they would not choose the same path although both easily agree that there is one short and steep path and one longer and flatter path.

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order. Since this requirement is unnaturally strong, the classical result that two vote-maximizing candidates choose the median location (Hotelling, 1929) stands apparently on highly questionable grounds. A way to avoid this issue would be to ask the participants in the poll not only about their favorite tax rate, but about a full ranking of the alternative tax rates. Apart from practical problems, the downside of such an approach is the informational requirement that political candidates know the full assessment of every voter. That is, we have replaced a questionable requirement by another one. A solution to this issue relates back to the seminal contribution of Black (1948). He examined single-peaked preferences on a line, which has the same effect as voters who are allowed to asses the "distances" between different tax rates individually. Black's result that under single-peaked preferences the median voter wins in majority voting against any other alternative has the following implication for the situation of spatial competition outlined above: in *any* location game that is consistent with the poll, both candidates choose the median tax rate in equilibrium. In that sense the classical result is *robust*.

The example on tax rates illustrates that in two-player location games on a line the questionable requirement of homogeneous distance perceptions is not driving the final outcome. However, for all other cases – in particular, for more than two players and for multi-dimensional spaces – robustness of the results is an open problem. If one can show that the model assumption is not driving the results, then the model is put on a solid foundation. This issue, although fundamental, seems to have been overlooked in the – rich and exciting – history of location games.

In this paper we want to scrutinize for given outcomes of spatial competition whether they rely on homogeneous distance perceptions or not. To this end, we formalize individual distance perceptions as individual edge lengths of a graph.[3] A formal description of consumers/voters of this type leads to a non-cooperative game between p players, which are interpreted as firms or political candidates. In this game, players simultaneously choose a location in order to maximize the number of agents (i.e., consumers/voters) they can attract. An equilibrium is then called *robust* if it is an equilibrium for all possible distance perceptions that are based on the same underlying structure (a line, for example). In other words, our modeling approach boils down to defining a stronger notion of equilibrium which we call *robust equilibrium*. It is defined directly on the situation of spatial competition, i.e., the underlying space and the distribution of agents (such as the poll on tax rates). Formally, several of location games correspond to the same situation of spatial competition, one for each setting of individual distance perceptions; and a robust equilibrium is a Nash equilibrium in any of these games. In particular, it is also a Nash equilibrium in the standard case of homogeneous distances.

A key result for our analysis is the characterization of robust equilibria by four conditions which are jointly necessary and sufficient. It is based on partitioning the underlying space into "hinterlands" and "competitive zones". Applying this result allows us first of all to judge which of the standard results are robust. In fact, we find that several outcomes do not depend on the assumption of homogeneous distances, but others do.

In the second part of the paper, we examine general properties of robust equilibria. Among them is the central issue of minimal differentiation (e.g., d'Aspremont, Gabszewicz, & Thisse, 1979; de Palma, Ginsburgh, Papageorgiou, & Thisse, 1985; de Palma, Hong, & Thisse, 1990; Eaton & Lipsey, 1975; Economides, 1986; Król, 2012; Meagher & Zauner, 2004). It turns out that robust equilibria satisfy a local variant of minimal differentiation, i.e., they induce reduced games in which the corresponding players are minimally differentiated. This result provides strong support for the "principle of minimal clustering" which has been proposed in the seminal contribution of Eaton and Lipsey (1975). Indeed, for any number of players, any underlying structure, and any distribution of agents, robust equilibria are characterized by clusters of players. That is, the players are jointly located on what we show to be the appropriately defined medians of local areas. Based on this result, we discuss the welfare implications for consumers and observe that almost all robust equilibria are not Pareto efficient. Consumers would unambiguously improve if some firm would be relocated appropriately. We finally, elaborate on the conditions for the existence of robust equilibria. We analyze how the spatial structure and the distribution of consumers/voters guarantee, admit, or preclude the existence of robust equilibria. Interestingly, two very common assumptions in the literature – (a) uniform distribution of consumers/voters and (b) one-dimensional space such as cycle or line structures – are mutually exclusive in the sense that for higher numbers of players robust equilibria require that one of them is not satisfied.

1.1. Related literature

There is an immense body of literature on spatial competition. While the original Hotelling–Downs framework is restricted to a one-dimensional space, a uniform distribution of agents, and only two players, many authors have attempted to relax these restrictions. To do so, one branch of the literature has followed a continuous modeling approach within the Euclidean space $\mathbb{R}^k$ (e.g., d'Aspremont et al., 1979; Economides, 1986), while a second branch replaces the Euclidean space by a graph (e.g., Labbé & Hakimi, 1991). Because the history of both branches is rich and long, providing a summary which covers all of it would exceed the scope of our paper. We restrict ourselves here to list several surveys on the topic and to discuss the most closely related works.

A broad overview and taxonomy of literature on spatial competition can be found in Eiselt, Laporte, and Thisse (1993). Based on five components (the underlying space, the number of players, the pricing policy, the rules of the game, and the behavior of the agents) the authors provide a bibliography for competitive location models. While this summary is not limited to certain subbranches, more specific surveys have been written on spatial models of consumer product spaces (Lancaster, 1990), on spatial competition in continuous space (Gabszewicz & Thisse, 1992), on spatial models of political competition (Mueller, 2003; Osborne, 1995), on competition in discrete location models (Plastria, 2001), on sequential competition (Eiselt & Laporte, 1997; Kress & Pesch, 2012), and on one-stage competition in location models (Eiselt & Marianov, 2011; ReVelle & Eiselt, 2005).

Although there are many variations and relaxations of spatial competition, virtually all of the models rely on the assumption of homogeneous distance perceptions. For instance, asymmetric transportation costs (e.g., Nilssen, 1997) do not alter the assumption. In order to examine to which extent this standard simplification is driving the results we will focus on the first stage of Hotelling's game, i.e., we will investigate the location choices of the players but we will not include additional variables such as prices. Similar approaches have been used, for example, by Eaton and Lipsey (1975), Denzau, Kats, and Slutsky (1985), and Braid (2005) who also concentrate on spatial competition by assuming fixed (and equal) prices. Nevertheless, extending our approach to a two-stage game would be a potential next step for further

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[3] This can be shown to be equivalent to the assumption of single-peaked preferences on certain domains. For example, if the underlying structure is a line graph, then this assumption is equivalent to the standard notion of single-peakedness. An alternative model variation would keep the assumption of homogenous distances but add a set of nodes (which we call "dummy nodes") to make the graph more flexible. As we show in Appendix B, this model variation would undermine the model's explanatory power.

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