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A multi-level Taguchi-factorial two-stage stochastic programming approach for characterization of parameter uncertainties and their interactions: An application to water resources management

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ABSTRACT

This paper presents a multi-level Taguchi-factorial two-stage stochastic programming (MTTSP) approach for supporting water resources management under parameter uncertainties and their interactions. MTTSP is capable of performing uncertainty analysis, policy analysis, factor screening, and interaction detection in a comprehensive and systematic way. A water resources management problem is used to demonstrate the applicability of the proposed approach. The results indicate that interval solutions can be generated for the objective function and decision variables, and a variety of decision alternatives can be obtained under different policy scenarios. The experimental data obtained from the Taguchi's orthogonal array design are helpful in identifying the significant factors affecting the total net benefit. Then the findings from the multi-level factorial experiment reveal the latent interactions among those important factors and their curvature effects on the model response. Such a sequential strategy of experimental designs is useful in analyzing the interactions for a large number of factors in a computationally efficient manner.

1. Introduction

The world has been turning its attention to the increasingly critical issue of water scarcity. According to the United Nations, approximately 700 million people in 43 countries are now suffering from water scarcity, and it is projected that 1.8 billion people will be living in countries or regions with absolute water scarcity by 2025 (UN-Water, 2006). The limited availability of water leads to a growing competition for water use among municipality, industry and agriculture in many countries. As rapid population growth and economic development, the competition for limited supplies will intensify, resulting in tensions and conflicts among water users. Therefore, wise decisions are desired to make best use of limited water resources. Optimization techniques have played an important role in helping decision makers (DMs) allocate and manage water resources in an effective and efficient way. However, a variety of uncertainties exist in water resources management systems and their latent interactions may further intensify the complexity in the decision-making process. As a result, conventional optimization methods such as linear programming, quadratic programming and integer programming would become ineffective when a variety of uncertainties exist in system components.

Over the past few years, a number of optimization methods have been proposed for dealing with uncertainties in water resources management (Abdelaziz, 2012; Bravo & Gonzalez, 2009; Chung, Lansey, & Bayraksan, 2009; Gaivoronski, Sechi, & Zuddas, 2012; Guo, Huang, Zhu, & Wang, 2010; Li, Huang, Nie, & Liu, 2008, 2009; Qin, Huang, Zeng, Chakma, & Huang, 2007; Teegavarapu, 2010; Wang & Huang, 2011, 2012). Among these methods, two-stage stochastic programming (TSP) has the ability to take corrective actions after a random event occurs (Birge & Louveaux, 1988, 1997). In a TSP model, two groups of decision variables can be distinguished. The first-stage decision must be made prior to the realization of random variables, and then the second-stage decision can be determined after a random event takes place. The recourse action in the second stage is effective in minimizing the risk of infeasibility as a result of the first-stage decision. TSP can thus be used to tackle uncertain information presented as probability distributions and make decisions in a two-stage fashion. However, TSP has difficulty in dealing with uncertainties when the sample size is too small to generate distribution functions. Even if such distributions are available, addressing them in large-scale optimization models can be challenging.

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Interval-parameter linear programming (ILP) is efficient in coping with uncertain information expressed as interval numbers with known lower and upper bounds but unknown distribution functions (Huang, Baetz, & Patry, 1992). Moreover, ILP can reflect interval information in the coefficients of the objective function and constraints, as well as in the solutions of the objective-function value and decision variables, which is helpful for DMs to interpret and adjust decision schemes according to practical situations. Consequently, an integration of TSP and ILP is desired to enhance the capability of addressing uncertainties in different formats (Huang & Loucks, 2000).

The aforementioned optimization methods mainly focus on addressing parameter uncertainties that exist in various formats such as intervals, fuzzy sets and probability distributions. However, they can hardly reveal the potential interactions among model parameters in the optimization model. It is thus necessary to explore the correlated parameters and their contributions to the variability of the model output. Factorial designs have been widely used to study the interaction effects of two or more factors on a response variable (Lewis & Dean, 2001; Lin, Huang, Lu, & He, 2008; Mabilia, Scipioni, Vegliò, & Tomasi Scianò, 2010; Onsekizoglu, Bahceci, & Acar, 2010; Qin, Huang, & Chakma, 2008; Wang & Huang, 2013; Wang, Huang, & Veawab, 2013; Zhou, Huang, & Yang, 2013). All these studies used the most popular two-level factorial design which assumed that the response was linear over the range of the factor levels chosen. However, many real-world problems involve the nonlinear relationships between the factors and the response. The two-level factorial experiment cannot address the nonlinear effects. Thus, the multi-level factorial design is proposed to detect the curvature in the response function (Box & Behnken, 1960; Wu & Hamada, 2009; Xu, Chen, & Wu, 2004). As the number of factors increases, the multi-level factorial design would become infeasible from a time and resource viewpoint due to a large number of experimental runs required.

To reduce the number of experiments to a practical level when there are many factors to be studied, factor screening is necessary to identify a few factors that have significant effects on the response and remove those insignificant ones at the early stage of the factorial experiment. The concept of Taguchi's orthogonal arrays is an effective and efficient means of identifying the importance of factors through performing only a small subset of the experimental runs (Adenso-Díaz & Laguna, 2006). Nevertheless, it can hardly provide information on how these factors interact. Thus, Taguchi's orthogonal arrays can be employed to screen out the important factors from a large number of potential factors in a computationally efficient way. Then the multi-level factorial design can be used to analyze the interactions among those important factors. Combining the Taguchi's orthogonal arrays with the multi-level factorial design is thus a sound strategy to study the potential interactions for a large number of factors at multiple levels.

The objective of this study is to develop a multi-level Taguchi-factorial two-stage stochastic programming (MTTSP) approach through incorporating ILP, TSP, Taguchi's orthogonal arrays, and the multi-level factorial design within a general framework. MTTSP is capable of analyzing parameter uncertainties and their interactions in a comprehensive and systematic manner. A water resources management problem will be used to illustrate the applicability of the proposed method.

2. Methodology

2.1. Interval-parameter two-stage stochastic programming

Consider a problem wherein a water manager is responsible for allocating water to multiple users, with the objective of maximizing the total net benefit through identifying optimized water-allocation schemes. As these users need to know how much water they can expect so as to make sound plans for their activities and investments, a prescribed amount of water is promised to each user according to local water management policies. If the promised water is delivered, it will bring net benefits to the local economy; otherwise, the users will have to obtain water from other sources or curtail their expansion plans, resulting in economic penalties (Maqsood, Huang, & Yeomans, 2005).

In this problem, a first-stage decision on the water-allocation targets must be made before unknown seasonal flows are realized. When the uncertainty of seasonal flows is uncovered, a second-stage recourse decision can be made to compensate for any adverse effects that may have been experienced as a result of the first-stage decision. Thus, this problem under consideration can be formulated as a TSP model (Huang & Loucks, 2000):

$$\text{Max } f = \sum_{i=1}^{m} NB_i T_i - E\left[\sum_{i=1}^{m} C_i S_{iQ}\right] \tag{1a}$$

subject to:

$$\sum_{i=1}^{m} (T_i - S_{iQ}) \leqslant Q, \tag{1b}$$

$$S_{iQ} \leqslant T_i \leqslant T_{i\max}, \ \forall i, \tag{1c}$$

$$S_{iQ} \geqslant 0, \ \forall i. \tag{1d}$$

where f is total net benefit ($); NB_i is net benefit to user i per meter^3 of water allocated ($/meter^3); T_i (first-stage decision variable) is allocation target for water that is promised to user i (meter^3); $E[\cdot]$ is expected value of a random variable; C_i is loss to user i per meter^3 of water not delivered, $C_i > NB_i$ ($/meter^3); S_{iQ} (second-stage decision variable) is shortage of water to user i when the seasonal flow is Q (meter^3); Q (random variable) is total amount of the seasonal flow (meter^3); $T_{i\max}$ is maximum allowable allocation amount for user i (meter^3); m is number of water users; i is index of water users, $i = 1, 2, 3$, with $i = 1$ for the municipality, $i = 2$ for the industrial sector, and $i = 3$ for the agricultural sector.

To solve the above problem through linear programming, the distribution of Q must be approximated by a set of discrete values (i.e. random seasonal flow can be discretized into three interval numbers representing low, medium and high flows with each having a probability of occurrence). Letting Q take values q_j with probabilities p_j ($j = 1, 2, \ldots, n$), we have:

$$E\left[\sum_{i=1}^{m} C_i S_{iQ}\right] = \sum_{i=1}^{m} C_i \left(\sum_{j=1}^{n} p_j S_{ij}\right) \tag{2}$$

Thus, model (1) can be reformulated as follows:

$$\text{Max } f = \sum_{i=1}^{m} NB_i T_i - \sum_{i=1}^{m}\sum_{j=1}^{n} p_j C_i S_{ij} \tag{3a}$$

subject to:

$$\sum_{i=1}^{m} (T_i - S_{ij}) \leqslant q_j, \ \forall j, \tag{3b}$$

$$S_{ij} \leqslant T_i \leqslant T_{i\max}, \ \forall i,j, \tag{3c}$$

$$S_{ij} \geqslant 0, \ \forall i,j. \tag{3d}$$

where S_{ij} denotes the amount by which the water-allocation target (T_i) is not met when the seasonal flow is q_j with probability p_j.

Model (3) is effective in tackling uncertainty in water availability (q_j) presented as probability distributions. However, uncertainties may also exist in other parameters such as net benefits (NB_i), penalties (C_i), and water-allocation targets (T_i). In real-world problems, it is difficult to generate probability distributions for