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Discrete Optimization

Efficient elementary and restricted non-elementary route pricing

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ABSTRACT

Column generation is involved in the current most efficient approaches to routing problems. Set partitioning formulations model routing problems by considering all possible routes and selecting a subset that visits all customers. These formulations often produce tight lower bounds and require column generation for their pricing step. The bounds in the resulting branch-and-price are tighter when elementary routes are considered, but this approach leads to a more difficult pricing problem. Balancing the pricing with route relaxations has become crucial for the efficiency of the branch-and-price for routing problems. Recently, the *ng*-routes relaxation was proposed as a compromise between elementary and non-elementary routes. The *ng*-routes are non-elementary routes with the restriction that when following a customer, the route is not allowed to visit another customer that was visited before if they belong to a dynamically computed set. The larger the size of these sets, the closer the *ng*-route is to an elementary route. This work presents an efficient pricing algorithm for *ng*-routes and extends this algorithm for elementary routes. Therefore, we address the Shortest Path Problem with Resource Constraint (SPPRC) and the Elementary Shortest Path Problem with Resource Constraint (ESPPRC). The proposed algorithm combines the Decremental State-Space Relaxation technique (DSSR) with completion bounds. We apply this algorithm for the Generalized Vehicle Routing Problem (GVRP) and for the Capacitated Vehicle Routing Problem (CVRP), demonstrating that it is able to price elementary routes for instances up to 200 customers, a result that doubles the size of the ESPPRC instances solved to date.

1. Introduction

Since the work of Christofides, Mingozzi, and Toth (1981) on the Capacitated Vehicle Routing Problem (CVRP), column generation has become a widely applied technique for exactly solving different routing problems. Currently, it is involved in almost all of the current most efficient approaches to routing problems. These approaches use integer and mixed-integer programming formulations with variables associated with the set of all possible routes. These formulations are set partitioning based, and these constraints impose the selection of the route to serve each customer. We refer to this as SPP formulation. The resolution of its linear relaxation requires the use of column generation techniques. The pricing subproblem to be solved is the Elementary Shortest Path Problem with Resource Constraints (ESPPRC).

The ESPPRC is a shortest path problem on a graph where the customers have an amount of resources that are consumed during a visit. The resource constraints require that the total of the resources consumed by any feasible solution does not exceed the existing limits. There may be edges with negative cost, and these edges may generate negative cycles, but because a feasible solution must be an elementary path, revisiting a customer is strictly forbidden.

The ESPPRC is a difficult to solve $\mathcal{NP}$-hard problem (Dror, 1994). In general, the current best performing algorithms have acceptable processing times when the optimal solution is a path with at most fourteen customers. We refer to Di Puglia Pugliese and Guerriero (2012) for a review of the approaches proposed throughout the last three decades. Most of the main ideas for the ESPPRC resolution can be found in their work and in the proceedings of Feillet, Dejax, Gendreau, and Gueguen (2004), Chabrier (2006), Righini and Salani (2006), Righini and Salani (2008) and Boland, Dethridge, and Dumitrescu (2006).

Instead of solving the ESPPRC, the original work of Christofides et al. (1981) solves its relaxation, the Shortest Path Problem with Resource Constraints (SPPRC). This relaxation does allow revisiting a same customer in a route. The resulting non-elementary routes

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are often called q-routes. However, the resource constraints are the same as the ESPPRC, and therefore, every time a customer is visited, the relevant resource consumption is counted, and the total consumption must still respect the existing limits. A recent survey about the SPPRC can be found in Di Puglia Pugliese and Guerriero (2013).

This relaxation has some interesting properties. First, as distinct from the original problem, it can be solved in pseudo-polynomial time using a dynamic programming algorithm. In addition, even relaxing the elementarity constraint, the SPP bounds found by its linear relaxation are usually strong, especially when there are few customers per route. Furthermore, to strengthen the bounds of the linear relaxation, Christofides et al. (1981) also demonstrated that size two cycles can be forbidden with almost no extra effort.

However, even with the 2-cycle elimination restriction, the linear relaxation is still weak, a behavior that motivated researchers to seek better cycle elimination devices to turn the non-elementary routes closer to elementary routes, without dealing with the whole complexity of the ESPPRC. The work of Irnich and Villeneuve (2006) devised an algorithm that solves the SPPRC by forbidding cycles of an arbitrary size. This algorithm is significantly more complicated, resulting in a complexity that grows factorially with the size of the cycles being forbidden. On account of this, their method quickly becomes impractical. Eliminating cycles of size four or more is already too time consuming compared with the bound improvement obtained. Such behavior was verified in practice by Fukasawa et al. (2006) for the CVRP.

Recently, Baldacci, Mingozzi, and Roberti (2011) proposed a compromise between routes and q-routes: ng-routes. These ng-routes are restricted non-elementary routes built accordingly to customer sets, ng-sets, which are associated with each customer and act like their "memory." Therefore, when a path reaches a given customer, it "forgets" whether another customer was visited if it does not belong to the ng-set of the current customer. Moreover, the further extension is only allowed to customers that are not "remembered". The ng-sets are usually composed of the closest customers and clearly with a larger size set the problem is harder to solve. This is due to the fact that the ng-routes generated are going to be increasingly closer to elementary routes.

Although the SPPRC with ng-routes can be solved in pseudo-polynomial time for a fixed ng-set size, to the best of our knowledge, there is no work that solves this problem for ng-sets larger than 20. This finding is observed because of the exponential-sized data structure used by Baldacci et al. (2011) to speed up the algorithm.

1.1. Contributions

This work aims at efficiently solving the SPPRC with restricted non-elementary routes and the ESPPRC. The first improvement is obtained by adapting the Decremental State-Space Relaxation (DSSR) technique of Righini and Salani (2008) to the SPPRC with ng-routes. This technique was initially proposed for the ESPPRC, where the elementarity restriction is relaxed and the problem is then solved iteratively, rebuilding the restrictions as needed, until the optimal solution is found. The main difference of our algorithm is instead of relaxing the elementarity of the routes, we relax the restriction imposed by the ng-sets.

Next, we accelerate this approach using completion bounds. Because an iteration of the DSSR is a relaxation for its next iteration, the completion bounds estimate lower bounds for completing the paths being built on a given iteration, and, given an upper bound for the optimal solution (which is usually equal to zero for pricing algorithms), it avoids the extension of paths that may exceed the known upper bound on the next DSSR iteration.

These two techniques were already used together in the work of Pecin (2010). In that work, a column generation procedure that uses the ESPPRC as the pricing subproblem was proposed for the CVRP. Using the algorithm of Fukasawa et al. (2006), instances with up to 100 customers could be solved to optimality pricing only elementary routes.

Finally, we demonstrate how our algorithm for the SPPRC with restricted non-elementary routes can be easily extended to generate only elementary routes. We also highlight the two new elements existing in our approach that allow us to double the size of the ESPPRC instances solved thus far.

The proposed algorithms are then applied to the Generalized Vehicle Routing Problem (GVRP), where the customers to be visited are clusters of vertices. Each cluster has an associated demand and can contain one or more vertices. The demand of each cluster must be fully collected in exactly one vertex of the cluster. This problem is a generalization of the CVRP and the Traveling Salesman Problem (TSP), and, as in the classical CVRP, identical vehicles are given, and routes must start and end at the depot and the capacity of the vehicle must not be exceeded. We report experiments demonstrating that our algorithms are able to solve the SPPRC with ng-set sizes up to 64 and the ESPPRC for hard instances of both the GVRP and CVRP. The CVRP instances are solved through reducing them to GVRP instances. The results of the column generation algorithm also provide a clear idea of the gains in the lower bounds comparing the SPPRC with different ng-set sizes and also with the ESPPRC, as well as the time required for computing them. In addition, several new best lower bounds are identified for the GVRP, especially for large instances.

This paper is organized as follows. Section 2 presents the ESPPRC and explains the required mathematical notation. The ng-route relaxation is described in Section 3. In Section 4, we explain the techniques used to solve the ng-route relaxation. In Section 5, we demonstrate how our algorithm can be used to obtain only elementary routes, and we highlight the main elements that allowed us to build a very efficient method for solving the ESPPRC. Section 6 presents the Generalized Vehicle Routing Problem formally. Section 7 reports the computational results for both the GVRP and CVRP. Finally, Section 8 presents some conclusions.

2. Elementary shortest path problem with resource constraints

Let $G = (V, A)$ be a graph with arc set A and vertex set V, which is composed of the set of customers $\mathcal{C}$ plus a source vertex s and a destination vertex t, and let $\mathcal{R}$ be a set of resources. For each arc $(i,j) \in A$, let c_{ij} be the cost of the arc and w_{ij}^r be the consumption of the edge, for each $r \in \mathcal{R}$. For each pair $i \in \mathcal{C}$ and $r \in \mathcal{R}$ let a_i^r and b_i^r be two non-negative values, such that the total resource consumption along a path from s to i must belong to the interval $[a_i^r, b_i^r]$. The ESPPRC aims to find a minimum cost elementary path from s to t that satisfies all resource constraints.

The resources constraints can model different types of restrictions. For instance, most vehicle routing problems consider that the vehicles have a known capacity, and this capacity cannot be exceeded in a single route. Other problems have time windows, which require the route to visit a customer in a given interval of time. Moreover, one can also view the elementarity constraint as resource constraints, where each customer defines a binary resource and when a route visits a customer, it consumes all of the associated resource.

In this work, we deal only with the capacity constraint, in addition to the obvious elementarity constraint. Thus, the customer set has an associated demand function $d : \mathcal{C} \to \mathbb{Z}$, and there is a global capacity limit Q, which no feasible solution may exceed. Because we apply our algorithm for routing problems, we can consider