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Decision Support

Risk averse decision making under catastrophic risk

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ABSTRACT

A nonstandard probabilistic setting for modeling of the risk of catastrophic events is presented. It allows random variables to take on infinitely large negative values with non-zero probability, which correspond to catastrophic consequences unmeasurable in monetary terms, e.g. loss of human lives. Thanks to this extension, the safety-first principle is proved to be consistent with traditional axioms on a preference relation, such as monotonicity, continuity, and risk aversion. Also, a *robust* preference relation is introduced, and an example of a monotone robust preference relation, sensitive to catastrophic events in the sense of Chichilnisky (2002), is provided. The suggested setting is demonstrated in evaluating nuclear power plant projects when the probability of a catastrophe is itself a random variable.

1. Introduction

The theory of choice under uncertainty aims to provide a coherent framework of principles of *rational behavior* for analyzing and guiding[1] decision maker's attitudes toward potential losses/rewards. It is traditionally studied on a space of lotteries or random variables (r.v.'s). While assessment of outcomes and corresponding probabilities of an r.v. ultimately depends on decision maker's preferences, does the resulting order (ranking) of r.v.'s adhere to principles of rational choice? For example, if an r.v. X_1 is preferred to an r.v. X_2 and the latter is preferred to an r.v. X_3, is X_1 preferred to X_3 (transitivity of a preference order)? The work of Von Neumann and Morgenstern (1953) is arguably the first fundamental study in the theory of choice that postulates axioms on a preference order: *completeness*, *transitivity*, *continuity*, and *independence* and shows that these four axioms admit a numerical representation in the form of expected utility function,[2] so that instead of verifying all four axioms, a decision maker merely needs to choose a utility function u and to rank given r.v.'s according to the expected value of u. Almost every decision theory views *risk aversion* as a cornerstone principle of rational behavior that states that given a choice between a random outcome X and a sure payoff equal to the expected value of X, a risk averse agent always prefers the latter. In the framework of the expected utility theory (EUT), risk aversion implies that the utility function u is concave and can be conveniently characterized by various measures through the derivatives of u, e.g. by the Arrow–Pratt measure of absolute risk-aversion $-u''/u'$. Using the ideas of Finetti (1937) and Von Neumann and Morgenstern (1953), Savage (1972) introduces somewhat similar four axioms[3] on a preference order: (a) transitivity and completeness (weak order), (b) the "sure-thing" principle, which parallels the independence axiom, (c) likelihood payoff independence (if an agent prefers getting a prize under event A rather than the same prize under event B, then this choice does not depend on the size of the prize), and (d) Archimedean axiom (agent preferences are robust with respect to low-probability events). Essentially, Savage's axioms replace objective probabilities by subjective ones, but still have a numerical representation in the form of expected utility. Hence, the resulting *subjective-probability expected utility theory* (SPEUT) retains most of the properties of the EUT and is often viewed as a version of the former. The intuitive appeal and mathematical simplicity of expected utility largely rests on the *independence* axiom ("sure-thing" principle), which implies linearity in probability. However, namely this axiom is widely acknowledged to contradict certain empirical/experimental evidence, commonly known as the Allais paradox[4] ("fanning out," common ratio and common consequence effects)

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[1] Individuals often correct their attitudes once inconsistencies of their choices are revealed to them; see Savage (1972).

[2] In fact, Bernoulli (1954) used the form of expected utility as early as in 1738 to explain famous St. Petersburg's paradox.

[3] In fact, Savage (1972) introduces seven axioms, but the other three are rather technical and have little relevance for the present discussion.

[4] Savage (1972) questions the validity of the Allais paradox: individuals usually correct themselves once they are shown that their choices fail to satisfy the independence axiom. This is where the theory of choice fulfills its educational role of being a guidance of rational behavior.

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(Allais, 1953; Machina, 1987). Similar to the Allais paradox, there is the Ellsberg paradox (*ambiguity aversion*) (Ellsberg, 1961) that contradicts the "sure-thing" principle. This has motivated either changing or completely omitting the independence axiom and, as a result, has given rise to a variety of so-called non-expected utility theories, including weighted expected utility theory (WEUT) (Fishburn, 1983), rank-dependent (anticipated) utility theory (AUT) (Quiggin, 1993), prospect theory (Fox & Poldrack, 2009; Kahneman & Tversky, 1979), cumulative prospect theory (CPT) (Kahneman & Tversky, 1992), regret theory (Loomes & Sugden, 1982), disappointment theory (Bell, 1985), dual utility theory (Röell, 1987; Yaari, 1987), etc.; see Machina (1987) for a detailed discussion of these and other theories. However, while resolving the Allais paradox, none of these theories are free from their own paradoxes. For example, in Yaari's theory (Yaari, 1987), a dual independence axiom, replacing the independence axiom, implies that a dual utility function linearly depends on outcomes and, thus, leads to paradoxes "dual" to the Allais paradox and the common ratio effect, in which the role of outcomes and probabilities is reversed. Among these theories, the CPT (Kahneman & Tversky, 1992) is arguably most sophisticated and the one that mimics individual's behavior most closely. It generalizes the prospect theory (Kahneman & Tversky, 1979) and Quiggin and Yaari's theories (Quiggin, 1993; Yaari, 1987) and prescribes a decision maker to use an *S*-shaped evaluation function for outcomes (convex for losses and concave for gains) and to transform the linear cumulative probability distribution as a function of probability into an inverse *S*-shape (unlikely but extreme outcomes become overweight, whereas outcomes in the middle of the distribution become underweight). Chateauneuf and Wakker (1999) show that for decision making under risk, the CPT is consistent with four axioms on a preference order: *weak ordering*, *continuity*, *stochastic dominance*, and *tradeoff consistency*. Remarkably, all the mentioned theories, whose title bears "utility," start from an axiomatic framework for a preference order and arrive at a corresponding utility function, whereas the prospect theory and the CPT originate from modeling of actual individual's behavior ("empirical realism") in terms of value function and probability and only then are "translated" into an axiomatic framework (Chateauneuf & Wakker, 1999). Nonetheless, the CPT is not immune to criticism: it assumes the existence of a constant (non-random) reference point interpreted typically as current endowment and, thus, fails to work when agent's endowment is uncertain, i.e., when the agent owns a stock and is thinking about selling/exchanging it. Also, the CPT postulates that for lotteries to be compared, agents should ignore lotteries' common outcomes ("isolation effect"), which resembles the independence axiom of the EUT and, thus, incurs similar criticism (Nwogugu, 2006).

While the discussed theories of choice display a steady progress toward understanding and modeling of individual's attitudes toward risk, there is a growing evidence questioning their applicability to decision making under *catastrophic risk* (Posner, 2005), which is characterized as rare events with extreme consequences, i.e. terrorist attacks, industrial accidents and natural/environmental disasters (floods, fires, earthquakes, oil spills, etc.). In fact, in all these theories, the axioms of rational behavior are designed from the perspective of a single investor (Grechuk, Molyboha, & Zabarankin, 2012) (or a group of investors (Jouini, Napp, & Nocetti, 2013; Nocetti, Jouini, & Napp, 2008) through *collective risk aversion*), whose goal is to attain gains beyond a risk-free return and who, if desired, may limit or completely eliminate exposure to risky assets. Those axioms may not be adequate for evaluating structural safety of construction projects that have limited ability to coup with catastrophic events (so-called "black swans"); see (Chichilnisky, 2002). For example, Buchholz and Schymura (2012) report that for low degrees of risk aversion, the EUT almost neglects catastrophic events, whereas for moderate levels of risk aversion, it leads to a "tyranny of catastrophic risk;" see also Ikefuji, Laeven, Magnus, and Muris (2010), Ackerman, Stanton, and Bueno (2010) and Weitzman (2009). Chichilnisky (2002, 2010) observes that for catastrophic events, neither the *continuity* axiom nor the traditional definition of *risk aversion* is applicable and introduces the "swan" axioms requiring subjective probabilities to be sensitive to both frequent and rare events. Moreover, the axioms of *continuity* and *risk aversion* are inconsistent with the *safety-first* principle (Chavas, 2004), whose objective is to minimize the probability of a catastrophic event. The existing literature on structural engineering and system safety offers several methods for estimating the severity and probability of catastrophic events (Banks, 2005), including the *extreme value theory*[5] (Coles, 2001; Novak, 2011; Embrechts et al., 1999) and *probabilistic safety assessment* (PSA) (also known as *probabilistic risk assessment* (PRA)) (Drouin et al., 2009), whereas actuarial mathematics has long employed the *Cramer–Lundberg model*[6] for estimating likelihood of rare events; see, e.g. Embrechts, Klüppelberg, and Mikosch (2012) and Norkin (2006). However, these methods are not intended for making the analysis of the estimated losses and probabilities to conform to the principles of rational choice. Moreover, Volkanovskia and Cepin (2011) noted that the PSA faces several uncertainties: in model, in parameters, and so-called completeness uncertainty. Using the PSA with Monte-Carlo simulation, they showed that because of those uncertainties, the *core damage frequency* (CDFr), characterizing the likelihood of damaging the core of a nuclear reactor and being a crucial safety characteristic for licensing nuclear power plants (Atomic Energy Agency, 2004; International Atomic Energy Agency, 2008; Nuclear Energy Agency, 2010), has a substantial dispersion and, in fact, should be treated as an r.v. with normal distribution. In economics, the idea that the probabilities of outcomes in question may themselves be unknown is often referred to as *Knightian uncertainty* (Knight, 1921) or *ambiguity*, whose significance remains a highly debated issue to this day (Arrow, 1951; Aven, 2011; Ellsberg, 1961; Runde, 1998).[7] For example, Klibanoff, Marinacci, and Mukerji (2005) and Nau (2006) propose axiomatic models of choice under uncertainty that generalize the EUT by allowing to distinguish and to incorporate attitudes toward both "risk" and "ambiguity" ("uncertainty"). In those models, a decision maker, still being an expected utility maximizer, may exhibit different degrees of risk aversion toward "risk" and "ambiguity." In economics and actuarial science, ambiguity (uncertainty in probability) is closely related to the notion of *self-protection* (Ehrlich & Becker, 1972).[8] In application to insurance problems, Alary, Gollier, and Treich (2013) use the model of Klibanoff et al. (2005) to show that ambiguity aversion, which could be associated with "more pessimism" under the SPEUT,

[5] The extreme value theory is a branch of statistics which deals with assessing probabilities of extremely rare events; see Coles (2001), Novak (2011) and Embrechts, Resnick, and Samorodnitsky (1999). To estimate the probability of catastrophic events with this theory, one needs to make assumptions about the tail distribution, which are usually hard to verify, see Embrechts (2000).

[6] In the ruin theory, the Cramer–Lundberg model, also known as the compound-Poisson risk model, is a closed-form formula for the probability of the ultimate ruin of an insurance company provided that customers' claims arrive according to a Poisson process. There is a generalization of this model for the case when inter-arrival time of claims has an arbitrary distribution.

[7] Knight (1921, p. 233) states that the difference between risk and uncertainty is that in the case of risk, the distribution of random outcomes is known, whereas in the case of uncertainty, it is not. This distinction, however, is a subject of extensive debate, see e.g. Arrow (1951), Ellsberg (1961), Runde (1998) and Aven (2011) and references therein.

[8] Self-protection is an investment/action that an agent makes to *reduce the probability* of an undesired event, e.g. getting vaccination, installing additional protective devices and security systems, reinforcing and fortifying structures, etc. However, in contrast to insurance, self-protection is often ignored for one of the reasons that in the case of such an event, it will be an additional loss. An infamous example of self-protection ignorance is the failure to reinforce and enlarge the levee system of New Orleans, LA, devastated by hurricane Katrina in August 29, 2005 mainly by flooding.