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Decision Support

Three-level warranty service contract among manufacturer, agent and customer: A game-theoretical approach

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ABSTRACT

A warranty is a service contract between a manufacturer and a customer which plays a vital role in many businesses and legal transactions. In this paper, various three-level service contracts will be presented among the following three participants; a manufacturer, an agent, and a customer. In order to obtain a better result, the interaction between the aforementioned participants will be modeled using the game theory approach. Under non-cooperative and semi-cooperative games, the optimal *sale price*, *warranty period* and *warranty price* for the manufacturer and the optimal *maintenance cost* or *repair cost* for the agent are obtained by maximizing their profits. The satisfaction of the customer is also maximized by being able to choose one of the suggested options from the manufacturer and the agent, based on the risk parameter. Several numerical examples and managerial insights are presented and used to illustrate the models presented in this paper.

1. Introduction

A warranty is a service contract attached to a product guaranteeing that the product fulfills certain, key commitments to the customer, examples of which include product reliability and failure-free operations for a specified period of time. The majority of customers prefer to purchase a product from a manufacturer with a warranty ensuring the replacement or repairing of the product during the warranty period, with little or no charge. A warranty is therefore an effective incentive for customers to purchase a product from a manufacturer with similar product quality and reliability to that of other manufacturers. Thus it does not come as a surprise that service providers usually offer a wide variety of service contracts to customers, ensuring an increase of sales leading to more profit. Customers, in turn, when faced with these choices of contract, can reap the benefits of selecting the one which best suits their needs.

In literature, warranties are categorized into several broad types (Murthy & Djamaludin, 2002). Free replacement as a kind of warranty policy is presented in many papers. In such a policy, the manufacturer replaces the failed item with a new one free of charge the first time, and repairs it at a low cost during the remainder of the warranty period (e.g. Rinsaka & Sandoh (2006), Zhou, Li, & Tang (2009) and Wu, Lin, & Chou (2006)). In some cases the manufacturer has to compensate the customer by refunding the price paid (money back warranty) in addition to free replacement (e.g. Boom (1998)).

Outsourcing services are considered as another type of warranty polices whereby when a product fails, an external agent to the supplier rectifies the failure using different options (e.g. Asgharizadeh & Murthy (2000), Murthy & Asgharizadeh (1996), Murthy & Yeung (1995), Jackson & Pascual (2008)).

Maintenance options in a service contract present the third type of warranty policy. For instance, some papers proposed models where customers could negotiate different maintenance options such as availability, reliability, and selecting time intervals between maintenance controls (e.g. Wang (2010), Maronick (2007)). Maintenance in lifetime warranty policies and corresponding models for predicting failures and estimating costs for such policies have been presented in some recent researches (e.g. Chattopadhyay & Rahman (2008)). In addition to the aforementioned research, Hartman and Laksana (2008) discussed some warranty contracts including restrictions on repairs and renewals. discussed some warranty contracts including restrictions on repairs and renewals. Recently, Shamsi et al. presented a three-partite service contract model regarding risk parameters and quantity discount policies (Shamsi, Esmaeili, & Monfared, 2013). They considered a penalty cost for the agent due to repair waiting times.

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A significant shortcoming of all the formerly detailed papers is that they have ignored the three-level interaction between the manufacturer, agent and customer. In other words, they have considered a two-partite interaction or a tri-partite Service Contract which lacked interaction between the three parties. This omission appeared anomalous considering that most products these days, particularly high-tech products, are supported by a third party (agent) warranty in addition to the manufacturer's warranty. For instance, huge companies like BMW,[1] Apple, etc. offer a third-party warranty instead of a dealership warranty or insured warranty. An additional common example is the iPad2 which is insured both by Apple Company[2] as the manufacturer and by Squaretrade[3] as the third-party (agent).

In this paper, several scenarios will be proposed between the manufacturer, agent and customer under different warranty options. The manufacturer offers two choices to the customer: (1) free replacement per failure during the warranty period, or (2) no warranty is offered. On the other hand, the agent offers three options to the customer: (1) repairing any failure of the product after the expiration of warranty for a fixed cost per failure, (2) repairing all failures which have occurred during the lifetime of the product for a fixed maintenance price, or (3) repairing the failed product at a fixed cost per failure during its lifetime. The last two options offered by the agent are contingent on the *manufacturer offering no warranty*. Therefore, the customer essentially faces three options; the first option of the manufacturer and the agent, the second option of the manufacturer and the agent and the second option of the manufacturer and the third option of the agent. Since the price of each service or product influences its sales volume, the sales volume of each suggested option based on its price will be addressed (Glickman & Berger, 1976). Thus, the customer investigates the manufacturer and agent's options based on the price of each option so as to maximize their own satisfaction. To enrich the model in question, regarding the relation between the satisfaction and the utility function of the customer, the risk parameter on the customer's side is also considered. Moreover, for each option, the optimal sale price, warranty price and warranty period for the manufacturer, and the optimal maintenance cost or repair cost for the agent will be determined by maximizing their profits. The interaction between the manufacturer, agent and customer is modeled using a game theory approach and this covers both non-cooperative and semi-cooperative games. In the non-cooperative game, two types of scenarios are considered. In the first scenario, Nash equilibrium is obtained while the manufacturer, agent and customer choose their strategies separately and simultaneously. In the second scenario, the manufacturer has more power than the agent (*Manufacturer Stackelberg* game) and the agent has more power than the customer (*Agent Stackelberg* game). Sub-game perfect equilibrium (SPE) is obtained by the backward induction method. In the semi-cooperative game, the manufacturer and the agent cooperate together and act as an integrated service-provider against the customer, thereby dominating the latter in a *Manufacturer–Agent Stackelberg* game.

The remainder of this paper is organized as follows. The notation and assumptions underlying the model will be given in Section 2. In Section 3, this problem will be formulated; including models for the manufacturer, agent and customer, and their optimal solutions under different warranty options will be discussed. Section 4 will present non-cooperative games (static and Stackelberg models) and a semi-cooperative game. Computational results including numerical examples and managerial insights illustrating the different models will be detailed in Section 5. Finally, the paper will be concluded in Section 6 with several suggestions for further research in this topic.

[1] https://www.bmw-warranty.co.uk/.
[2] http://www.apple.com/support/products/.
[3] http://www.apple.com/support/products/.

2. Notation and assumptions

This section introduces the notation of the models. In addition, all decision variables, input parameters and assumptions which will be used to formulate the models will be stated.

2.1. Decision variables

P_{ip}	sale price received per unit item by the manufacturer under option $i, i = 1, 2$
T_w	warranty period offered by the manufacturer
P_w	warranty price per unit item paid by the customer to the manufacturer
P_a	maintenance price announced by the agent to the customer
C_{ir}	repair cost charged by the agent to the customer per unit item under option i

2.2. Input parameters

L	lifetime of product
C_p	production cost per unit item
P_s	salvage value of a failed product $(P_s < C_p)$
C'_r	repair cost per unit item incurred by the agent
N_1	number of failures during the lifetime of product
N_2	number of failures during the warranty period
N_3	number of failures after warranty has expired
X_i	operational time of product after the ith repair and before the $(i+1)$th failure
$\widetilde{X}$	operational time of product after the last repair and before termination of its lifetime
$\lambda(t)$	failure rate
r	rate of ageing of product
S_i	sales volume under option i
K_0	an amplitude constant factor $(K_0 > 0)$
α_{ji}	price elasticity of manufacturer and agent $(j = M, A)$ under option i; $(\alpha_{ji} > 1)$
γ_i	risk parameter of the customer under option $i; i = 1, 2, 3$
R	quantitative satisfaction of customer under each option
Π_{Mi}	profit of manufacturer under option $i; i = 1, 2$
Π_{Ai}	profit of agent under option $i; i = 1, 2, 3$
Π_{SP}	profit of service provider
Π_{Ci}	total satisfaction of customer under option $i; i = 1, 2, 3$
$U_i(C)$	utility function of customer under option $i, i = 1, 2, 3$

2.3. Assumptions

The proposed models are based on the following assumptions:

1. There is one customer, one agent and one manufacturer.
2. The failure intensity is an increasing function of time and is based on the model proposed by Jackson and Pascual (2008). In this model, the failure hazard, $\lambda(t)$, is given by