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Interfaces with Other Disciplines

Efficient firm groups: Allocative efficiency in cooperative games

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ABSTRACT

The concept of *efficiency in groups* postulates that a coalition of firms has to record a smaller distance toward the aggregate technology frontier compared with the sum of individual distances. Efficiency analysis (either allocative or technical) is defined with respect to cooperative *firm game* in order to provide operational distance functions, the so-called pseudo-distance functions. These pseudo-distances belong to the core interior of the allocative firm game, in other terms, any given firm coalition may always improve its allocative efficiency. We prove that such a result is impossible for technical efficiency, *i.e.*, the technical efficiency cannot increase for all possible coalitions.

1. Introduction

The aggregation of indexes, scores or utility functions is an important feature in many fields such as microeconomics and the social choice theory as well. Gorman's overlapping theorem (1968), based on additive separability, is the corner stone of many developments in welfare economics (see Blackorby, Donaldson, & Auersperg, 1981; Blackorby & Donaldson, 1982) and in production economics (see Blackorby et al., 1978). Along this line, the aggregation of the Debreu–Farrell efficiency index is investigated by Färe and Lovell (1988). The aggregation of industry efficiency, *i.e.* for a group of firms, is analyzed by Färe, Grosskopf, and Li (1992), Li (1995), and Li and Ng (1995), and also by Zelenyuk (2006) in the Malmquist index case, among others.

An impossibility result is outlined by Blackorby and Russell (1999). Any efficiency measure related to a group of firms cannot be decomposed into the sum of each firm's individual efficiency. This impossibility is partly solved in welfare economics by use cooperative games approaches, precisely by the application of Shapley's (1953) value (Shorrocks, 1999, 2013) and the Nested Shapley Value (Chantreuil & Trannoy, 1999, 2013). Mussard and Peypoch (2006) investigate Owen's value for the measurement of industry productivity. The authors bring out a Malmquist productivity index decomposition for a group of firms in which the contribution (the imputation) of each firm to the overall amount of the industry productivity is captured. Those results, relying on cooperative games and on specific values, provide a partial aggregation. The imputation is not necessarily grounded on the same axiomatic shape compared to the efficiency (productivity) index to be decomposed. In some cases, the sum of the contributions of each firm is not equal to the overall productivity: the axiom of aggregate efficiency used in cooperative games is sometimes violated.

Another line of research, not investigated in this paper, is devoted to the measurement of productivity and efficiency (super-efficiency) with respect to DEA (data envelopment analysis). The concept of super-efficiency is first introduced by Andersen and Petersen (1993) in DEA frameworks. The decision making unit (DMU), which is under evaluation, is temporarily excluded from the data set in order to determine whether or not this unit improves the score of efficiency, *i.e.*, the performance of the firm.[2] When one DMU under evaluation is excluded from the firm, we retrieve the idea of marginal contribution used in cooperative games. Lozano (2012) introduces the link between cooperative games and DEA models in order to show that the organizations may take benefit from cooperation when they share data about inputs and outputs. Lozano (2013) also introduces production games in which different organizations have the possibility to merge. Accordingly, two scenarios are itemized. The organizations join a coalition either by using their own technologies or by merging their technologies. Both production games are balanced so that the core of the game is non-empty. The organizations improve their profit when

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[2] This technique has been further explored in DEA models aiming at detecting outliers (see among others Banker & Chang, 2006) or aiming at detecting infeasibilities (see for instance the discussion between Chen, 2005, and Soleimani-damaneh, Jahanshahloo, & Foroughi, 2006).

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they take benefit from a common and joint technology.

In the present paper, we follow Lozano (2013) without exploring DEA models. We keep the idea that the cooperation may improve allocative efficiency. The notion of super-efficiency has been associated with DEA models about the performance of DMUs, whereas it seems also natural to connect it to the measurement of allocative and technical efficiencies of industries. We introduce *efficiency in groups* on the basis of aggregate technologies, *i.e.*, technologies of firm coalitions. In our framework, the efficiency in groups is embodied by the cooperation between firms, for which either allocative or technical efficiency may be improved.[3] The performance of a firm is gauged by excluding temporarily that group from all possible firm coalitions. The concept of efficiency in groups is related to *super-efficiency* in the sense that the underlying game is *concave*.

We first recognize that the work introduced by Briec, Dervaux, and Leleu (2003), about the aggregation of directional distance functions, may be generalized. In spite of the impossibility result outlined by Blackorby and Russell (1999), Briec et al. (2003) introduce the way to compute directional distance functions for a group of firms (structural technical efficiency), and accordingly, they find necessary and sufficient conditions to maintain an exact aggregation, *i.e.*, the structural technical efficiency is decomposable into the sum of individual technical efficiencies associated with each firm. If the necessary and sufficient conditions (such as identical firm technologies) are not invoked, Blackorby and Russell's (1999) result holds: the exact aggregation is impossible and a positive aggregation bias arises. The distance of the firm group is always greater or equal to the sum of the distances related to each firm. This result is proven independently by Färe, Grosskopf, and Zelenyuk (2008).

In our model, the structural technical efficiency is computed by assuming that the firm (group) under evaluation is temporarily excluded from all possible firm coalitions it may belong to. The building-block pattern of our approach is then the possibility to consider coalitions within a cooperative game layout, namely a *firm game*, in which the concept of value enables pseudo-distance functions to be deduced. We prove that the firm game is always super-additive, and as a consequence, the aggregation bias is positive: firm coalitions cannot improve their technical efficiency. On the contrary, in *allocative firm games*, the result is reversed: the game is sub-additive and the allocative bias is negative. Hence, firm coalitions record a better allocative efficiency compared with the sum of their individual allocative efficiencies. The cooperation between firms always improves allocative efficiency. The result is attractive because the solution derived from the allocative firm game is in the core interior of the game if the allocative bias is submodular. This solution is called pseudo-allocative distance function since it satisfies the property of linear homogeneity in the same manner as the allocative distance function. We also show that the exact aggregation condition provides an unstable solution since it is not located in the core interior of the firm game. Finally, an *allocative fixed frontier game* is introduced in order to capture the role of the technical efficiency when the information about the technologies of all coalitions is not complete. Comparing the imputations of the allocative firm game with that of the allocative fixed frontier game enables some trade-off to be discussed.

The remainder of the paper is organized as follows. Section 2 introduces the following definitions: the directional distance function, the technology, and the aggregation bias for a group of firms. In Section 3, we examine cooperative firm games, the notion of efficiency in groups, and the concepts of core and imputation. In Section 4, we prove that firm games cannot yield an increase of technical efficiency for all firm coalitions. In Section 5, we show that allocative firm game provides efficient firm groups, since the allocative efficiency can always be improved. The solution of the allocative firm game, the so-called pseudo-allocative distance function, is investigated in Section 6. In Section 7, a discussion outlines the trade-off between allocative and technical efficiency improvements. Section 8 closes the article and exposes several possible extensions.

2. Technology, directional distance functions and aggregation bias

A production technology describes how inputs $x = (x_1, \cdots, x_n) \in \mathbb{R}^n_+$ are transformed into outputs $y = (y_1, \ldots, y_m) \in \mathbb{R}^m_+$. The production possibility set T is the set of all feasible input and output vectors:

$$T = \left\{(x, y) \in \mathbb{R}^{n+m}_+ : x \text{ can produce } y\right\}.$$

It satisfies the following assumptions[4]

(T1): $(0, 0) \in T, (0, y) \in T \Rightarrow y = 0$ *i.e.*, no free lunch;
(T2): the set $A(x) = \{(u, y) \in T : u \leqslant x\}$ of dominating observations is bounded $\forall x \in \mathbb{R}^n_+$, *i.e.*, infinite outputs cannot be obtained from a finite input vector;
(T3): T is closed;
(T4): $\forall z = (x, y) \in T, (x, -y) \leqslant (u, -v) \Rightarrow (u, v) \in T$, *i.e.*, fewer outputs can always be produced with more inputs, and inversely;
(T5): T is convex;
(T6): $\forall \lambda \geqslant 0, (x, y) \in T \Rightarrow (\lambda x, \lambda y) \in T$ (constant returns to scale).

The directional distance function introduced by Chambers, Chung, and Färe (1996, 1998) [5] $D_T : \mathbb{R}^{n+m}_+ \times \mathbb{R}^{n+m}_+ \longrightarrow \mathbb{R}_+$ involving a simultaneous input and output variation in the direction of a preassigned vector $g = (g_i, g_o) \in -\mathbb{R}^n_+ \times \mathbb{R}^m_+$ is defined as:

$$D_T(x, y; g) = \sup_\delta \{\delta \in \mathbb{R} : (x - \delta g_i, y + \delta g_o) \in T\}.$$

In the sequel, we do not investigate the cases of infeasibilities for which $D_T(x, y; g) = -\infty$, the directional distance being such that $D_T(x, y; g) \geqslant 0$.

We analyze the behavior of $\mathcal{K}$, a group of $|\mathcal{K}|$ firms with technology T^k, where $k \in \mathcal{K} := \{1, \ldots, |\mathcal{K}|\}$. Briec et al. (2003) define the aggregation bias as follows:

$$AB(\mathcal{K}; g) \ := \ D_T\left(\sum_{k\in\mathcal{K}} (x^k, y^k); g\right) - \sum_{k\in\mathcal{K}} D_{T^k}(x^k, y^k; g).$$

It provides the loss of technical efficiency due to the cooperation between the firms of group $\mathcal{K}$. If the aggregation bias is nil, the exact aggregation condition is:

$$D_T\left(\sum_{k\in\mathcal{K}} (x^k, y^k); g\right) = \sum_{k\in\mathcal{K}} D_{T^k}(x^k, y^k; g).$$

Briec et al. (2003) show under (T1)–(T4) that the aggregation bias is nil if the technologies are identical and when the input set is one-dimensional. Under the additional assumptions (T5)–(T6), the bias is nil if the firms use the same technique. We will see that those results do not tell us the whole story about the potential efficiency that the firms could capture when they form some coalitions. For that purpose, we study the aggregate technology of any firm coalition within a cooperative game framework.

[3] We examine efficiency in groups without invoking DEA although our approach could be adapted to DEA when the number of firms is not too important (see Deng & Papadimitriou, 1994).

[4] See Shephard (1970) for the analysis of their implications on technology.

[5] See also Chambers and Färe (1998), Chambers (2002), and Färe and Primont (2006) for more details on directional distance functions.