European Journal of Operational Research 234 (2014) 411–421

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Robust portfolios that do not tilt factor exposure

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ARTICLE INFO

Article history:
Available online 27 March 2013

Keywords:
Investment analysis
Robust portfolio model
Robustness analysis
Fundamental factors

ABSTRACT

Robust portfolios reduce the uncertainty in portfolio performance. In particular, the worst-case optimization approach is based on the Markowitz model and form portfolios that are more robust compared to mean–variance portfolios. However, since the robust formulation finds a different portfolio from the optimal mean–variance portfolio, the two portfolios may have dissimilar levels of factor exposure. In most cases, investors need a portfolio that is not only robust but also has a desired level of dependency on factor movement for managing the total portfolio risk. Therefore, we introduce new robust formulations that allow investors to control the factor exposure of portfolios. Empirical analysis shows that the robust portfolios from the proposed formulations are more robust than the classical mean–variance approach with comparable levels of exposure on fundamental factors.

1. Introduction

Robust optimization was developed to solve problems where there is uncertainty in the decision environment, and therefore is sometimes referred to as uncertain optimization (Ben-Tal and Nemirovski, 1998). Robust models were adapted in portfolio optimization to resolve the sensitivity issue of the mean–variance model (Markowitz, 1952) to its inputs. Even though the Markowitz model is still the basis for portfolio optimization and one of the most significant contributions to portfolio selection, its drawbacks are well documented. For example, Best and Grauer (1991) show that mean–variance portfolio weights are extremely sensitive to changes in asset means when no constraints are imposed. Broadie (1993) points out errors in computing efficient frontiers and stresses that estimates of mean returns using historical data should be used with caution. Chopra and Ziemba (1993) go one step further to analyze the relative impact of estimation errors in means, variances, and covariances.

These studies naturally led to the advancement of robust portfolio optimization. Costa and Paiva (2002) optimize robust tracking-error optimization when the expected returns and covariance matrix are not exactly known. Similarly under uncertain situations, El Ghaoui et al. (2003) solve the worst-case value-at-risk problem when only bounds on the inputs are known. In addition, Goldfarb and Iyengar (2003) introduce a way to formulate robust optimization problems as second-order cone programs using ellipsoidal uncertainty sets, and similar approaches are investigated by Tütüncü and Koenig (2004). As summarized, much effort has been put into defining uncertainty in input parameters and formulating the problems using worst-case optimization methods.[1]

On the same topic but on a different track, there have been recent developments looking into the behavior of robust portfolios formed from worst-case optimization (Kim et al., accepted for publication-a; Kim et al., 2012; Kim et al., 2013). They look for unexpected properties of robust formulations in order to not only expand our understanding on robust portfolios but also to increase its practical use. Focusing on the worst-case formulation with ellipsoidal uncertainty sets, Kim et al. (2012) mathematically show how robustness leads to higher correlation with fundamental factors. Kim et al. (accepted for publication-a) further investigate this behavior using worst-case formulations with ellipsoidal and box uncertainty sets under various settings to confirm the relationship between robustness and increased correlation with factors. Even though some worst-case formulations may result in portfolios with unintended factor loadings, robust portfolios are still required to protect portfolio returns from unexpected market movements. Kim et al. (2013) find that under the existence of market regimes, a portfolio that focuses on asset returns during the worst regime of the stock market results in a portfolio that is not only robust but has an improved performance based on measures such as Sharpe ratio, drawdown, and value-at-risk.

Based on these research findings, we know that investors can achieve robustness through robust optimization techniques but at the same time lose control of the factor dependency of the optimal portfolio. Factors explain the underlying movement of the

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[1] For further contributions on robust portfolio optimization, refer to Fabozzi et al. (2007a,b), Fabozzi et al. (2010), and Kim et al. (accepted for publication-b).

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http://dx.doi.org/10.1016/j.ejor.2013.03.029

market and widely studied factors include the market index, size (market capitalization), and book-to-market ratio factors (Fama and French, 1993). In addition to these three fundamental factors, portfolio managers look for macroeconomic factors and also identify statistical factors through principal component analysis. Investors and portfolio managers need to control the factor exposure for managing the portion of total risk generated from each factor because it allows them to better understand the overall risk of their portfolios.[2] Therefore, in this paper, we introduce formulations that form robust portfolios which are not tilted towards factors. Investors may consider robust counterparts of mean–variance portfolios for achieving robust performance. However, if the factor exposure of the robust counterparts is affected by the robust formulations, investors need to compare not only the performance between the classical mean–variance and robust portfolios but also their factor exposures. By forming portfolios from our proposed optimization problems, investors will be able to control the risk associated with each factor of robust portfolios. In other words, investors can hold portfolios that are robust and at the same time have a better understanding of how the portfolio will be affected by movements in market factors.

Before we introduce and analyze the revised robust formulations, we need a measure to compare the robustness of portfolios. This robustness measure allows us to compare the optimal portfolio from the new formulations with the existing mean–variance and robust portfolios. Therefore, we begin our analysis by defining a robustness measure and exploring its properties. Next, we derive new robust formulations to form optimal portfolios that match a target factor exposure by adding constraints to the original robust formulation. Since the additional constraints are developed based on factor exposure measures of portfolios, the new formulations will not only add robustness but also keep the factor dependency to a specified level. Furthermore, we perform empirical tests to confirm the factor exposures of the new robust portfolios and find that the new approach forms portfolios that are more robust than Markowitz portfolios without affecting the factor exposure.

The organization of the paper is as follows. Section 2 defines the robustness measure and its properties, and the new robust formulations that do not affect factor exposure are introduced in Section 3. Section 4 includes empirical tests using industry-level returns to confirm our development, and Section 5 concludes.

2. Robustness measure

In this section, we define a robustness measure prior to developing new robust formulations. By defining a measure for portfolio robustness, investors can compare the robustness among portfolios and also use this measure to set the desired robustness level when choosing an optimal portfolio. Once we define a robustness measure, we analyze its properties for the case when the geometry of the uncertainty set for the expected asset returns is an ellipsoid.

2.1. Defining a robustness measure

We begin defining a robustness measure by identifying the meaning of robustness. In general, $X(\varepsilon)$, which is a function of an uncertain parameter ε, being robust means that the value of $X(\varepsilon)$ is relatively stable even when the uncertain parameter ε fluctuates. Therefore, we can measure the robustness of $X(\varepsilon)$ by the movement of $X(\varepsilon)$ with respect to ε.

The robustness in portfolio management can be defined in similar fashion. We first determine a suitable uncertain function $X(\varepsilon)$ for measuring portfolio robustness. Since the optimal portfolio weights are a function of uncertain parameters μ and Σ, expected return and covariance of returns, respectively, the weights of the optimal portfolio are one possible choice for the function $X(\varepsilon)$. However, this measure does not reflect the portfolio robustness desired by investors because investors want robust portfolio performance but not necessarily robust portfolio weights. In other words, the stability of the portfolio performance is more important to investors than the stability of the optimal weights. Thus, it is more reasonable to use the portfolio performance for $X(\varepsilon)$ when we define portfolio robustness. Consequently, we can say that the optimal portfolio ω^* is robust if the performance of the portfolio ω^* is stable with respect to μ and Σ.

Based on the above definition of robustness in portfolio optimization, we derive the robustness measure for portfolios. The stability level of portfolio performance determines the degree of robustness. Therefore, the robustness measure can be defined by the performance fluctuation from the change in uncertain parameters. Portfolios that are robust will have small performance fluctuations and therefore low levels for the robustness measure. For simplicity, we assume that the only uncertain parameter is the mean of asset returns μ and the uncertainty is revealed at time T. Since the portfolio performance depends on the mean of asset returns μ and the portfolio weight ω^*, it can be represented as $f(\omega^*, \mu)$. The expected value of μ at time 0 is μ_0 and the realized value at time T is μ_T. Hence, investors make investment decisions based on μ_0 at time 0 but they evaluate the portfolio performance using μ_T at time T. Thus, performance fluctuation is the difference between the expectation and the realization of portfolio performance,

$$d(\omega^*) = |f(\omega^*, \mu_0) - f(\omega^*, \mu_T)|.$$

However, we cannot evaluate $d(\omega^*)$ at time 0 because μ_T is unknown until time T. So, instead of $d(\omega^*)$, the maximum of $d(\omega^*)$ on the uncertainty set U of μ_T should be the robustness measure,

$$D(\omega^*) = \max_{\mu_T \in U} d(\omega^*) = \max_{\mu_T \in U} |f(\omega^*, \mu_0) - f(\omega^*, \mu_T)|.$$

When the realization μ_T of the uncertain parameter μ varies in a given set U, the fluctuation of the portfolio performance cannot exceed the robustness measure $D(\omega^*)$. It follows that portfolios with smaller values of $D(\omega^*)$ will have less fluctuation in portfolio performance and can be considered to be more robust.

The generalized form of the robustness measure is

$$\text{Robustness measure} = \max_{\theta \in \Xi_0} |f(\omega^*, \bar{\theta}) - f(\omega^*, \theta)|,$$

- $f(\omega, \theta)$: portfolio performance,
- ω^*: an optimal portfolio,
- θ: uncertain parameters,
- $\bar{\theta}$: the expected value of uncertain parameters,
- Ξ_0: an unit uncertainty set for θ.

2.2. Properties of the robustness measure

For the robustness measure of portfolio performance, we assume the uncertain parameter μ to be in an uncertainty set U. Since ellipsoidal uncertainty sets are often used in robust optimization, we also assume that the uncertainty set of μ to be in an ellipsoid. The robust portfolio formulation with an ellipsoidal uncertainty set is,

$$(R_\delta) \quad \min_{\omega \in \Omega} \max_{\mu \in U_\delta(\bar{\mu})} \frac{1}{2}\omega'\Sigma\omega - \lambda\mu'\omega = \min_{\omega \in \Omega} \frac{1}{2}\omega'\Sigma\omega - \lambda\bar{\mu}'\omega + \lambda\delta\sqrt{\omega'\Sigma_\mu\omega},$$

where $\Omega = \{\omega | \omega'\iota = 1\}$, $U_\delta(\bar{\mu}) = \left\{\mu | (\mu - \bar{\mu})'\Sigma_\mu^{-1}(\mu - \bar{\mu}) \leqslant \delta^2\right\}$, where Σ is the covariance of expected returns, ι is a vector of ones,

[2] The importance of managing systematic risk through factors is illustrated, for example, by Fung and Hsieh (1997), Clark et al. (2002), Alexander and Dimitriu (2004), and Zhu et al. (2011).