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Dynamic asset allocation for varied financial markets under regime switching framework

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ABSTRACT

Asset allocation among diverse financial markets is essential for investors especially under situations such as the financial crisis of 2008. Portfolio optimization is the most developed method to examine the optimal decision for asset allocation. We employ the hidden Markov model to identify regimes in varied financial markets; a regime switching model gives multiple distributions and this information can convert the static mean–variance model into an optimization problem under uncertainty, which is the case for unobservable market regimes. We construct a stochastic program to optimize portfolios under the regime switching framework and use scenario generation to mathematically formulate the optimization problem. In addition, we build a simple example for a pension fund and examine the behavior of the optimal solution over time by using a rolling-horizon simulation. We conclude that the regime information helps portfolios avoid risk during left-tail events.

1. Introduction

Portfolio optimization is the most researched and practiced method to examine the optimal decision for asset allocation. The mean–variance model introduced by Markowitz (1952) is the basis for portfolio selection, which finds the optimal portfolio by computing the risk-return tradeoff using the estimated mean vector and the covariance matrix of asset returns. One of the advantages of the Markowitz model is that there are no restrictions on the type of assets that can be included in the model. For example, commodity futures, which has become drastically popular among investors as a major asset class, like stocks and bonds can be easily included in the model to solve the portfolio selection problem. However, the Markowitz model is a single-period model without stochastic characteristics and also assumes that the multi-dimensional return series of assets have constant mean vector and covariance matrix.

In this paper, we extend the traditional Markowitz portfolio model to address the changing nature of the covariance matrix under differing market conditions. Certainly, one of the severe issues arising during 2008 crash was the increase in correlation (contagion) that occurred and the ensuing lack of diversification by many investors (even those applying Markowitz models). The regime detection methodology provides an intuitive and practical way to anticipate changing correlation conditions. As such, the research is on the pathway of the original Markowitz tradition. There have been many studies indicating the existence of multiple *regimes* in financial markets especially the stock market. The hidden Markov model (HMM) is a popular method for regime identification, which has been widely used in engineering and science. Hamilton (1989) uses HMM to predict business cycles of the US economy by analyzing the US Gross National Product (GNP). Further discussions on HMM in finance can be found in Turner et al. (1989), Hansen (1992), Hamilton and Susmel (1994), and Garcia (1998). These researches commonly describe that the high (low) return regime of the equity market shows low (high) volatility. In addition, Guidolin and Timmermann (2007, 2008) identify four regimes in the joint return series of the stock market and the bond market by using HMM.

In our study, we construct a regime switching model that includes the commodity market index as well as the stock and bond indices by applying HMM. The main reason for including the commodity market as an additional asset class is because commodities are popular among practitioners for diversifying their portfolios. In addition, there are a number of academic studies that treat commodities as a financial asset class. For instance, Gorton et al. (2007) argue the commodity futures prices are proxies of commodity spot prices. In addition, Gorton and Rouwenhorst (2006) assert that the return of the commodity futures market is negatively correlated with the return of the equity market while positively correlated with inflation. This model allows us to estimate the mean vector and the covariance matrix of the joint return series described above for each regime. In other words, the returns of the three indices are stochastically emitted from one of many possible distributions.

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Incorporating a regime switching model relaxes the assumption of a single distribution of the Markowitz model. Instead of a fixed value for the expected return and variance, a set of probabilities representing the likelihood in each state is available. This probabilistic information converts the static mean–variance model into an optimization problem under uncertainty, which is the case for a market with unobservable multiple regimes. We take the stochastic programming approach to formulate this optimization problem. A reasonable extension of the mean–variance model is the n-period stochastic programs because they are theoretically equivalent to static problems (Ziemba, 2009). Birge and Louveaux (1997) and Ruszczyński and Shapiro (2003) provide a comprehensive introduction to stochastic programming. We examine the effect of asset allocation under a regime switching environment through a simple example for a pension fund with a goal of achieving a certain level of financial wealth within a limited time span.

The organization of the paper is as follows. Section 2 introduces HMM for analyzing market regimes, and the empirical results along with the interpretation of the identified regimes are documented in Section 3. Section 4 includes a simple example of stochastic optimization under the regime switching framework. Then, we conduct rolling-horizon simulations to construct stochastically optimized active portfolios and compare their performance with several benchmarks in Sections 5, and 6 concludes.

2. Regime identification

While many studies on financial regime identification using HMM focus heavily on the stock market, there is, to the best of our knowledge, no empirical research specifically on the regimes of the bond market. We suspect that a main reason is because there are only a few recognizable crises in the bond market prior to the European debt crisis of 2011. For the commodity market, Cheung and Miu (2010) study the diversification benefit of commodities by using Gorton's equal-weighted portfolio (Gorton and Rouwenhorst, 2006). They investigate commodity futures and find evidence that the diversification benefits are robust over time and across regimes. Moreover, they conclude that the high (low) return environment for commodity futures is also associated with low (high) volatility. However, their work incorporates neither the correlation between the stock and bond markets nor the relationship between the stock and commodity futures markets. Prajogo (2011) applies HMM to the two-dimensional return series composed of the S&P500 and the agricultural sector in the US stock market. Although she does not use the information of correlation between the two return series, the model provides the estimated covariance matrix for each regime and therefore allows the investors to identify the characteristics of correlation in each regime.

2.1. Data

The choice of representative indices is based on important characteristics of each market. For example, in the equities market, the S&P500 index is widely used in academic research and investment planning as a proxy for the market portfolio described in the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965). As for the bond market, since one of the major characteristics of bonds as a financial instrument is its stability, we gather the yield-to-maturity on 10-year benchmark US government bonds. Furthermore, we utilize the Goldman-Sachs Commodity Index for the commodity market because of its dominant popularity in the financial industry.[2] The time span of data is from January 2, 1980 to June 11, 2012, which covers over 30 years of trading history including the recent financial crisis.

[2] The data for all three markets are retrieved from Datastream where the identifiers are TOTMKUS (RI), BMUS10Y (RI) and GSCIEXR, respectively.

2.2. Hidden Markov model

The construction of HMM requires decisions on two characteristics of the model. The first one is the probability density function of the observations. In this paper, we assume that the three return series that each represents the stock, bond, and commodity markets, respectively, follow a multivariate normal distribution and this allows us to easily include correlations among financial markets in our model. Student's t distribution is known to be more robust to outliers for pattern recognition in gesture or speech using HMM (Chatzis et al., 2009). The reason for using the normal distribution instead of the Student's t distribution in our study, however, is because extreme tail events in financial markets cannot simply be modeled as noise or errors; these extreme events are essential for identifying market status such as the bubble period in the early 2000s or the market crash in 2008. In fact, it is known that the hidden Markov model with the normal distribution can address relatively fat tailed distributions properly. See Mulvey and Zhao (2010, Working Paper) for further discussion. The second property is the structure of the underlying hidden states that emit observable price series. We cannot directly observe market states and only the index information for each market is available. In our study, we assume that the number of hidden states is discrete and finite.

Fraser (2008) documents the basic framework of HMM. For ease of understanding, we follow his notations and derivations on HMM,

- $S(t)$: A random variable of (unobservable) state at time t
- $Y(t)$: A random variable of observation (in this case, three-dimensional daily return series of stock, bond, and commodity markets) at time t
- S_{t_1,t_2}: A sequence of random variables of states from time t_1 to t_2
- Y_{t_1,t_2}: A sequence of random variables of observations from time t_1 to t_2
- $s(t)$: A realized (unobservable) state at time t
- $y(t)$: A realized observation at time t
- s_{t_1,t_2}: A sequence of realized states from time t_1 to t_2
- y_{t_1,t_2}: A sequence of realized observations from time t_1 to t_2
- Θ: A set of variables of HMM parameters to be estimated
- θ: A set of estimated HMM parameters
- N: The dimension of observations (in this case, $N = 3$)
- $t \in \{1,\ldots,T\}$, $\forall s(t) \in S = \{1,\ldots,K\}$

We assume that the daily return series of the representative indices for the three markets are the observations of the model and the returns at time t depend on the regime at that time. We can write our model by using Fraser's notation as,

$$Y(t)|S(t) = y(t)|s(t) \sim N(\mu_{s(t)}, \Sigma_{s(t)}),$$

where $\mu_{s(t)}$ denotes the mean of the daily return series, $\Sigma_{s(t)}$ is the covariance matrix under state $s(t)$. The assumption of normality of returns and finite number of states allow the Baum–Welch algorithm to estimate model parameters. The Baum–Welch algorithm is an expectation–maximization (EM) algorithm for applications of HMM.[3] Given the initial set of parameters θ and the realized series of observations $y_{1,T}$, the solution of this algorithm always converges to a local maximum of the likelihood function $P_\theta(y_{1,T})$. We use randomly selected initial parameters and repeat the parameter estimation for each seed. We generate initial parameters with the following steps. First, calculate mean μ_i and volatility σ_i for index i. Next, we

[3] Please see Fraser (2008) for a detailed mathematical derivation on the algorithm.