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Tri-criterion inverse portfolio optimization with application to socially responsible mutual funds

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ABSTRACT

We present a framework for inverse optimization in a Markowitz portfolio model that is extended to include a third criterion. The third criterion causes the traditional nondominated frontier to become a surface. Until recently, it had not been possible to compute such a surface. But by using a new method that is able to generate the nondominated surfaces of tri-criterion portfolio selection problems, we are able to compute via inverse optimization the implied risk tolerances of given funds that pursue an additional objective beyond risk and return. In applying this capability to a broad sample of conventional and socially responsible (SR) mutual funds, we find that there appears to be no significant evidence that social responsibility issues, after the screening stage, are further taken into account in the asset allocation process, which is a result that is likely to be different from what many SR investors would expect.

1. Introduction

In the seminal work of Markowitz (1952, 1956) and later in books (1959, 1987, 2000) only the expected returns and the covariances of the returns of all considered assets are taken into account when attempting to locate optimal portfolios. However, recent studies suggest that in many situations a more complex decision model may be at work (Abdelaziz, Aouni, & Fayedh, 2007; Ballestero, Bravo, Pérez-Gladish, Arenas-Parra, & Plà-Santamaria, 2012; Bollen, 2007; Dorfleitner, Leidl, & Reeder, 2012; Dorfleitner & Utz, 2012; Hallerbach, Ning, Soppe, & Spronk, 2004; Steuer, Qi, & Hirschberger, 2007; Xidonas, Mavrotas, Krintas, Psarras, & Zopounidis, 2012). Toward that end, in Hirschberger, Steuer, Utz, Wimmer, and Qi (2013), a methodology is developed for extending the portfolio selection model of Markowitz to include a third criterion. Whether the third criterion is a financial or, as in this paper, a nonfinancial one, this causes the nondominated frontier[1] in two-dimensional space to become a nondominated surface in three-dimensional space, and that paper describes an algorithm for computing tri-criterion nondominated surfaces exactly. In this paper, going one step beyond, we utilize information generated by the algorithm for tri-criterion nondominated surfaces in an *inverse portfolio optimization* context, and the contributions of this paper are twofold.

One is that we show how inverse portfolio optimization can be used in a tri-criterion model. In bi-criterion mean–variance portfolio selection, inverse portfolio optimization is described by Zagst and Pöschik (2008). In their description, one first computes the entire nondominated frontier, for instance with the critical line method of Markowitz (1956), and then notes that each portfolio along the nondominated frontier has its own risk tolerance (or risk aversion) parameter. The goal is to compute the *implied* risk tolerances of given portfolios (which are likely not to be on the nondominated frontier) by matching them with close-by portfolios that are on the nondominated frontier and then considering those portfolios' risk tolerances as proxies for the risk tolerances of the given portfolios.

A fundamental input for the inverse portfolio optimizations conducted in this paper is a nondominated surface. Here, our paper relates to Xidonas and Mavrotas (2013), who use an ε-constraint method (see Haimes, Lasdon, & Wismer, 1971) to generate a discretized representation of a nondominated surface. Because accurate knowledge of the nondominated surface is important in inverse optimization, we use the approach of Hirschberger et al. (2013) which enables us to compute the entire nondominated surface exactly, rather than being consigned to working only with a collection of dispersed dots.

The second contribution consists of an empirical part, where we apply inverse portfolio optimization in a tri-criterion framework to socially responsible (SR) mutual funds. Here, our aim is to understand whether the term "socially responsible" is merely a sales

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[1] Markowitz (1952) calls this the *efficient frontier*, but we prefer the term *nondominated frontier*.

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pitch or whether fund managers in fact really take social responsibility into account throughout the whole investment process.

A socially responsible investing setting is appropriate for studying, in addition to financial criteria, a non-financial (third) criterion for two reasons. Firstly, the amounts already invested in SR mutual funds point to the demand for such products. Thus, there are clearly investors with further preferences besides financial ones. Secondly, with there being agencies that rate the socially responsible efforts of firms, studies can be conducted to investigate the ways SR mutual funds do or do not actually incorporate social responsibility into their operations.

We examine a broad sample of conventional and SR mutual funds and use ESG-scores from the Thomson Reuters ASSET4 database. These ESG-scores (where ESG stands for "environment, social, and governance") are assessments of firms' efforts to satisfy standards with respect to social responsibility as in the AA1000 AccountAbility Principles. A typical ESG-score is computed as follows. A firm is rated on a number of issues such as employment quality, health and safety, human rights, product responsibility, emissions, board composition, and so forth. Finally, the ratings are aggregated in such a way that the best firm in the rating universe gets a score of 100%, and the worst gets a score of 0.

The results of the empirical part in this paper relate to the recent literature comparing the performance of conventional (or unscreened) mutual funds to SR mutual funds or screened portfolios in general (Bello, 2005; Guerard, 1997; Hamilton, Jo, & Statman, 1993). In particular, we cannot confirm that conventional mutual funds exhibit superior financial performance. Moreover, in contrast to Hamilton et al. (1993), we find that conventional mutual funds tend of have, if anything, a higher portfolio return volatility. Although the screening process leads to fewer opportunities for diversification and hence a smaller feasible region in decision space, we find that SR investors do not have to accept significantly higher risk. This indicates that socially responsible firms can be less prone to earnings shocks.

However, comparing only financial performance ignores that investors may gain additional utility by specifically investing in socially responsible companies. In general, this additional utility stems from higher ESG fund scores. Commonly, SR funds follow a two-stage process. In the first stage, they filter out firms that do not meet their specific requirements regarding social responsibility (screening process). In the second (asset allocation) stage the fund's total wealth is allocated across the remaining assets. Somewhat surprisingly, we find that SR mutual funds do not exhibit significantly higher ESG-scores than their conventional counterparts. Using inverse portfolio optimization, we also find that SR mutual funds' managers are not too anxious to give up financial performance in favor of higher ESG-scores in the second stage.

The paper is organized as follows. The tri-criterion model, which produces a surface as opposed to a frontier, is theoretically introduced in Section 2. This is followed by an explanation of our inverse portfolio optimization process in Section 3. We explain our data in Section 4. Section 5 enumerates the hypotheses tested. Results are discussed in Section 6, and with final remarks, the paper concludes in Section 7.

2. Model

We now describe the model used in our study. Starting with a general von Neumann and Morgenstern (1947) utility function u, the expected utility, as shown by Pratt (1964), of a portfolio with random portfolio return R_P can be computed as

$$\boldsymbol{E}[u(R_P)] = u(\mu_P) + \frac{1}{2}u''(\mu_P)\sigma_P^2 + o(\sigma_P^2) \tag{1}$$

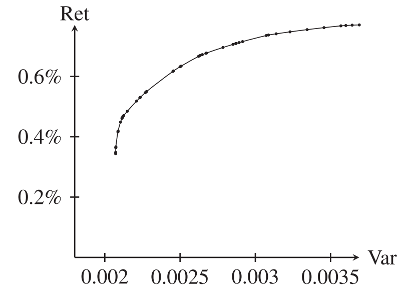

Fig. 1. A bi-criterion nondominated frontier. It is composed of a connected collection of parabolic segments. The frontier shown is from a bi-criterion problem with $n = 50$ securities and has 46 segments.

where μ_P denotes expected portfolio return and σ_P^2 denotes the variance of portfolio return. The residual term $o(\sigma_P^2)$ is of smaller order than σ_P^2. Following common practice, we go along with Feldstein (1969), Tobin (1969), Tsiang (1972), Bierwag (1974), Levy (1974), and Chamberlain (1983) who show that under the assumption that the investor's utility function is quadratic or that security returns follow a multinormal distribution, maximizing (1) is *equivalent* to maximizing

$$\widehat{\Psi}(\mu_P, \sigma_P^2, A) = -\frac{1}{2}A\sigma_P^2 + \mu_P, \tag{2}$$

where $A = -u''(\mu_P)/u'(\mu_P)$ is Arrow's absolute risk aversion (Arrow, 1965). By "equivalent" we mean that maximizing (1) and (2) yield the same solutions. Substituting $\lambda_\mu = 2/A$ and multiplying (2) by λ_μ, we have[2]

$$\Psi(\mu_P, \sigma_P^2, \lambda_\mu) = -\sigma_P^2 + \lambda_\mu\mu_P. \tag{3}$$

In (3), λ_μ represents the risk tolerance of the investor regarding expected return. Varying the risk tolerance parameter λ_μ over the nonnegative portion of the real line and maximizing Ψ causes expected utility to yield the Markowitz nondominated frontier in variance-expected return space as in Fig. 1. It is to be noted that the nondominated frontier is segment-wise parabolic. That is, it is made up of a connected collection of parabolic segments. The dots in Fig. 1 along the nondominated frontier are where the different parabolic segments connect with one another.

Since expected utility Ψ cannot explain why certain investors would specifically choose SR mutual funds, it is suggested by Bollen (2007) and Derwall, Koedijk, and Horst (2011, and references therein) that these investors also obtain utility from the social component of their investments. Furthermore, Ballestero et al. (2012) develop a financial-ethical model to select optimal portfolios for SR funds based on the three criteria of expected return, variance of return, and risk aversion. They use the concept of stochastic goal programming (Ballestero, 2001) to incorporate an ethical goal into the framework of classical utility theory under uncertainty.

As discussed earlier, we confine the general Bernoulli decision theory setting (1) to a mean–variance setup (3). Here, expected portfolio return is calculated as

$$\mu_P = \boldsymbol{\mu}^T\boldsymbol{x}$$

and portfolio variance is calculated as

$$\sigma_P^2 = \boldsymbol{x}^T\boldsymbol{\Sigma}\boldsymbol{x},$$

[2] As a technical note, this transformation requires $u'' \neq 0$. However, for risk-averse investors, $u'' < 0$ always holds due to the concavity of u.