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Discrete Optimization

Optimal single machine scheduling of products with components and changeover cost

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ABSTRACT

We consider the problem of scheduling products with components on a single machine, where changeovers incur fixed costs. The objective is to minimize the weighted sum of total flow time and changeover cost. We provide properties of optimal solutions and develop an explicit characterization of optimal sequences, while showing that this characterization has recurrent properties. Our structural results have interesting implications for practitioners, primarily that the structure of optimal sequences is robust to changes in demand.

1. Introduction

The problem of scheduling products with changeovers on a common resource, like a single machine, exists in many manufacturing settings. For instance, a machine processes different types of circuit boards by inserting components (cf. Coffman, Nozari, & Yannakakis, 1989), or a paper machine is capable of producing different grades of paper (cf. Magnanti & Vachani, 1990). One common feature of these problems is that products of the same type are often grouped to form batches. A changeover is incurred whenever the machine is switched over to process a different batch of products. The resulting sequencing and batching decisions are then made based on a number of possible criteria, including for example, total flow time, total changeover cost, number of late products, etc.

This paper considers a setting where a manufacturer fabricates multiple types of components on a single machine that are ultimately assembled into identical products. A fixed cost is incurred whenever there is a changeover from one component to another. A component is immediately available for assembly after its completion. Our objective is to determine an optimal sequence (involving both sequencing and batching decisions) that minimizes the weighted sum of total flow time and changeover cost.

We show that there exists a series of recurrent optimal sequences, and we obtain closed-form solutions for the integer batch sizes in these sequences. Based on the proven properties and the existence of the recurrent optimal sequences, we provide an explicit characterization of optimal sequences for any size problem. We also show that when there is a changeover cost, rather than a changeover time, the optimal sequence exhibits a constant-batch-size pattern. This result is especially interesting given that the algorithms of Coffman et al. (1989), who consider the same setting but use changeover time rather than cost, lead to non-increasing batch sizes in the optimal sequences.

The rest of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we introduce the formulation of the problem. Section 4 describes the major results about the properties and a characterization of optimal sequences. Managerial implications of our solution are discussed in Section 5.

2. Literature review

There has been extensive research on changeover scheduling problems with batching; we refer the reader to the surveys by Potts and Kovalyov (2000) and Allahverdi, Ng, Cheng, and Kovalyov (2008). Many of these papers refer to setups instead of changeovers, although both terms can be used interchangeably. To avoid confusion and maintain consistency, we use the term changeover throughout the paper.

The existing literature on changeover scheduling with batching can be divided into two main streams depending on whether or not

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the products are composed of components of multiple types. The first research stream studies batching and/or sequencing decisions for products without components. Most of the work in this stream is conducted under a single machine setting. For instance, Santos and Magazine (1985) and Dobson, Karmarkar, and Rummel (1987) independently started this research stream by proposing the same single-machine batching problem to minimize the total flow time of one product. In both papers, a constant changeover time is needed before the machine can process a different batch of products. Dobson et al. (1987) also provide heuristics for the multi-product case. Cheng and Kovalyov (2001) consider a single-machine scheduling problem for multiple products with deadlines. A changeover cost is incurred when the machine is switched to produce another product. The objective is to minimize total changeover costs. A few papers address more complex manufacturing environments. Cheng, Kovalyov, and Chakhlevich (2004) and Lin and Cheng (2005), for example, minimize the makespan of a two-stage flowshop. The two papers differ in how they model the second stage of production (dedicated machines or single flexible machine) and where batching operations are performed (at the first stage or the second stage).

The second research stream, which is more relevant to our work, focuses on scheduling products with multiple component types. A product is not completed until all of its components are finished, which adds to the complexity of the problem. One set of papers in this stream considers the problem of processing components on parallel dedicated machines. In some cases, there is a subsequent assembly stage, in which all of the components are assembled into a final product. This problem is sometimes referred to as the assembly scheduling problem with batching operations. For instance, Kovalyov, Potts, and Strusevich (2004) study a two-stage assembly scheduling problem minimizing the makespan. In other cases, the assembly stage is ignored. For instance, Lin and Cheng (2011) consider a batch scheduling model in a concurrent open shop where operations of each product are independently performed on their parallel dedicated machines. They consider minimizing the maximum lateness, weighted number of tardy products and total weighted completion time. In this paper, we focus on the problem of scheduling products with components on a single bottleneck machine. We assume that there is ample capacity at other resources, such as for example an assembly stage, and we thus do not explicitly capture these non-bottleneck resources in our model.

A second set of papers in this stream study the scheduling of products with multiple component types on a single flexible machine. Papers in this set assume each product consists of two components; one component is common to all products and the other one is unique to each product. The assumption of changeover time leads to a batching of the common components, but has no effect on the unique components. Baker (1988) and Vickson, Magazine, and Santos (1993) consider the same batching problem on a single machine to minimize flow time, but use different assumptions regarding the calculation of flow time for each product. Gerodimos, Glass, and Potts (2000) and Lin (2002) study the same model as Baker (1988) and Vickson et al. (1993) but consider two due-date related objectives, minimizing the maximum lateness and the number of late products. These papers also differ in how flow time is computed. A similar model for a more complex environment is studied by Cheng and Wang (1999) where they show minimizing the makespan for a two-stage flowshop is NP-complete.

Finally, a third set of papers study the scheduling of products with multiple component types, all of which are common to all products. This is the single machine problem we address in this paper. Both batching and sequencing decisions are needed for these different common components and thus this problem is quite complex. Coffman et al. (1989) started this research stream by considering the problem of scheduling identical products with two common component types on a single machine to minimize flow time. A fixed changeover time is incurred whenever the machine is switched to process a batch of components of a different type. They provide efficient algorithms to obtain optimal sequences. Cheng, Ng, and Yuan (2008) consider a model of scheduling products with multiple operations on a single machine with the objective of minimizing the total completion time. The operations of each product belong to different families. Each family of operations can be grouped as a batch which requires a changeover time. They provide some complexity results for the problem and show that some special cases are solvable in polynomial time.

Most papers of these research streams assume changeovers incur a time delay, but a few papers do consider the case where changeovers could result in costs. As pointed out by Magnanti and Vachani (1990), considering changeover cost instead of time may be more appropriate in some settings, where changeovers can be executed after the regular production hours without consuming resource time or when changeover time is small.

To the best of our knowledge, only Kella (1991) and Gim and Han (1997) have studied the problem of scheduling products with multiple common components on a single machine taking into account changeover cost. Both of these papers assume an infinite horizon problem setting. Kella (1991) studies a setting where a machine sequentially produces two types of fluid components which are mixed to produce a final product. A fixed changeover cost is incurred whenever the machine is switched from producing one type of fluid component to the other. An optimal cyclic policy is provided to minimize the long-run average cost that includes both inventory holding cost and changeover cost. Gim and Han (1997) address a variant of this problem by assuming N different types of components are processed on a single machine and then assembled into identical products. Both changeover time and cost are incurred when the machine is switched to produce a different component. They develop a heuristic based on a repetitive sequence for an infinite horizon problem to minimize total production cost, which consists of changeover cost, inventory cost and other costs. Our model can be viewed as a discrete optimization version of Kella (1991). While Kella (1991) studies an infinite horizon continuous optimization problem in which there is no sequencing decision, we derive solutions for both the finite and the infinite horizon problem, including the sequencing of the components.

Our finding that there exists a series of recurrent optimal sequences is consistent with the optimality of a stationary periodic policy suggested by Kella (1991). We find that when there is a changeover cost, rather than a changeover time, the optimal sequence exhibits a constant-batch-size pattern. This structural insight of non-increasing optimal batch sizes has also been supported in Santos and Magazine (1985), Dobson et al. (1987), Coffman and Yannakakis (1990), Mosheiov and Oron (2008), all of which consider changeover time. This result is especially interesting given that the algorithms of Coffman et al. (1989), who consider the same setting but use changeover time rather than cost, lead to non-increasing batch sizes in the optimal sequences.

3. Model framework

There are n identical products that need to be produced on a single machine. While our structural results and related insights extend to the case with more than two components (see Appendix B), for expositional clarity throughout the main part of the paper, we assume that each product is composed of two components. Specifically, each product requires one component of type A with processing time $a > 0$ and one component of type B with processing time $b > 0$. For convenience, define $T = b/a$. Without loss of